

# Secure Smart Contracts with Isabelle/Solidity<sup>1</sup>

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Joint work with Asad Ahmed, Achim D. Brucker, Naipeng Dong, Horacio Mijail,  
Billy Thornton, and Mark Utting

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<sup>1</sup>Supported by the Engineering and Physical Sciences Research Council [EP/X027619/1]

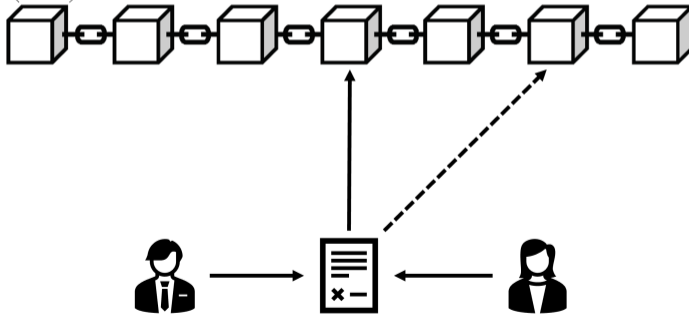
# Smart Contracts

from	to	amount
0x5B3...	0xAb8...	5378
0x4B2...	0x787...	2782
...	...	...



# Smart Contracts

from	to	amount
0x5B3...	0xAb8...	5378
0x4B2...	0x787...	2782
...	...	...



```
contract Bank {  
    mapping(address => uint256) balances;  
  
    function deposit() public payable {  
        balances[msg.sender] = balances[msg.sender] + msg.value;  
    }  
  
    function withdraw() public {  
        uint256 bal = balances[msg.sender];  
        balances[msg.sender] = 0;  
        msg.sender.transfer(bal);  
    }  
}
```

### Solidity

```
contract Customer {
  Bank bank;

  constructor(Bank b) public {
    bank = b;
  }
  function deposit(uint v) public {
    bank.deposit.value(v)();
  }
  function withdraw() public {
    bank.withdraw();
  }
  function() external payable {
    //received some funds
  }
}
```

### Solidity

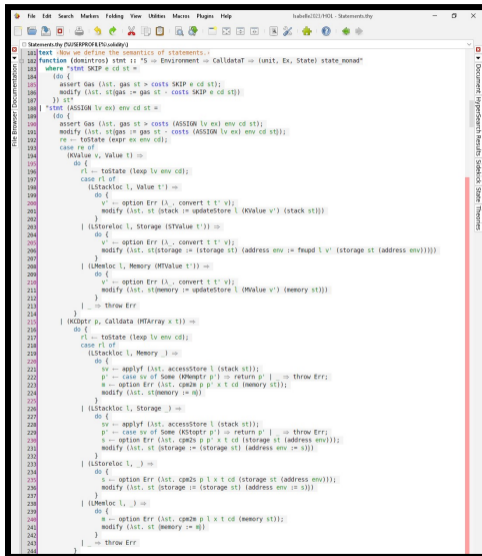
```
contract Bank {
  mapping(address => uint256) balances;

  function deposit() public payable {
    balances[msg.sender] =
    balances[msg.sender] + msg.value;
  }

  function withdraw() public {
    uint256 bal = balances[msg.sender];
    balances[msg.sender] = 0;
    msg.sender.transfer(bal);
  }
}
```

It is estimated that since 2019,  
more than \$5B was stolen  
due to vulnerabilities in smart contracts

Isabelle/Solidity is a deep empedding of Solidity (v0.5.16) in Isabelle/HOL



```
182 text "Now we define the semantics of statements."
183 function (domintros) st0 :: 'S ⇒ Environment ⇒ Calldata ⇒ (unit, Ex, State) state_monad"
184 where "st0 SKIP e cd st =
185   (do {
186     assert Gas (Ast.gas st > costs SKIP e cd st);
187     modify (λst. st.gas := gas st - costs SKIP e cd st);
188     }) st"
189 "st0 (ASSIGN lv ex) env cd st =
190   (do {
191     assert Gas (Ast.gas st > costs (ASSIGN lv ex) env cd st);
192     modify (λst. st.gas := gas st - costs (ASSIGN lv ex) env cd st);
193     re ← toState (expr ex env cd);
194     case re of
195     | (KValue v, Value t) =>
196       do {
197         r1 ← toState (Lexp lv env cd);
198         case r1 of
199         | (LStackLoc l, Value t') =>
200           do {
201             v' ← option Err (λ_. convert t t' v);
202             modify (λst. st (stack := updateStore l (KValue v') (stack st)));
203           }
204         | (LStoreLoc l, Storage (SValue t')) =>
205           do {
206             v' ← option Err (λ_. convert t t' v);
207             modify (λst. st (storage := (storage st) (address env := fupd l v' (storage st (address env)))));
208           }
209         | (LMemLoc l, Memory (MValue t')) =>
210           do {
211             v' ← option Err (λ_. convert t t' v);
212             modify (λst. st (memory := updateStore l (MValue v') (memory st)));
213             | _ => throw Err
214           }
215         | (KCDptr p, Calldata (MArray x t)) =>
216           do {
217             r1 ← toState (Lexp lv env cd);
218             case r1 of
219             | (LStackLoc l, Memory _) =>
220               do {
221                 sv ← applyf (λst. accessStore l (stack st));
222                 p' ← case sv of Some (KMemptr p') => return p' | _ => throw Err;
223                 m ← option Err (λst. cm2m p p' x t cd (memory st));
224                 modify (λst. st (memory := m));
225               }
226             | (LStackLoc l, Storage _) =>
227               do {
228                 sv ← applyf (λst. accessStore l (stack st));
229                 p' ← case sv of Some (KStgptr p') => return p' | _ => throw Err;
230                 s ← option Err (λst. cm2s p p' x t cd (storage st (address env)));
231                 modify (λst. st (storage := (storage st) (address env := s)));
232               }
233             | (LStoreLoc l, _) =>
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242               }
243             | _ => throw Err
244           }
245         )
246       }
```



## Introduction

Smart Contracts  
Solidity

## Isabelle/Solidity

Problem

Isabelle/Solidity

## Conclusion

Applications  
Summary

Isabelle/Solidity is a deep embedding of Solidity (v0.5.16) in Isabelle/HOL

- *Fixed-size integer types* with and without overflow.

```
182 text "How we define the semantics of statements."
183 function (doIntro) stmt :: 'S ⇒ Environment ⇒ Calldata ⇒ (unit, Ex, State) state_monad"
184 where "stmt SKIP e cd st =
185   (do {
186     assert Gas (Ast. gas st > costs SKIP e cd st);
187     modify (λst. stGas := gas st - costs SKIP e cd st);
188   }) st"
189 "stmt (ASSIGN lv ex) env cd st =
190   (do {
191     assert Gas (Ast. gas st > costs (ASSIGN lv ex) env cd st);
192     modify (λst. stGas := gas st - costs (ASSIGN lv ex) env cd st);
193     re ← toState (expr ex env cd);
194     case re of
195     | (MValue v, Value t) =>
196       do {
197         r1 ← toState (Lexp lv env cd);
198         case r1 of
199         | (LStackLoc l, Value t') =>
200           do {
201             v' ← option Err (λ_. convert t t' v);
202             modify (λst. st (stack := updateStore l (KValue v') (stack st)));
203           }
204         | (LStoreLoc l, Storage (SValue t')) =>
205           do {
206             v' ← option Err (λ_. convert t t' v);
207             modify (λst. st (storage := (storage st) (address env := fupd l v' (storage st (address env)))));
208           }
209         | (LMemLoc l, Memory (MValue t')) =>
210           do {
211             v' ← option Err (λ_. convert t t' v);
212             modify (λst. st (memory := updateStore l (MValue v') (memory st)));
213           }
214         | _ => throw Err
215       }
216     | (KCDptr p, Calldata (MArray x t)) =>
217       do {
218         r1 ← toState (Lexp lv env cd);
219         case r1 of
220         | (LStackLoc l, Memory _) =>
221           do {
222             sv ← applyf (λst. accessStore l (stack st));
223             p' ← case sv of Some (KMemptr p') => return p' | _ => throw Err;
224             m ← option Err (λst. cm2m p p' x t cd (memory st));
225             modify (λst. st (memory := m));
226           }
227         | (LStackLoc l, Storage _) =>
228           do {
229             sv ← applyf (λst. accessStore l (stack st));
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238           }
239         | (LMemLoc l, _) =>
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242             modify (λst. st (memory := m));
243           }
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245       }
246     )
```



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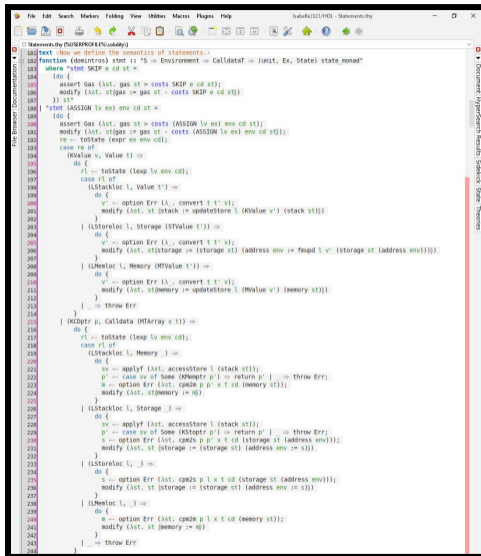
- *Fixed-size integer types* with and without overflow.
- *Domain-specific primitives*, such as transfer or balance.

```
182 text >How we define the semantics of statements.>
183 function (domintros) stmt :: 'S => Environment => Calldata => (unit, Ex, State) state_monad*
184 where *stmt SKIP e cd st =>
185 (do {
186   assert Gas (Ast. gas st > costs SKIP e cd st);
187   modify (Ast. stgas := gas st - costs SKIP e cd st);
188   }) st
189 *state (ASSIGN lv ex) env cd st =>
190 (do {
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192   modify (Ast. stgas := gas st - costs (ASSIGN lv ex) env cd st);
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195     (MValue v, Value t) =>
196       do {
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198         case r1 of
199           (LStackloc l, Value t') =>
200             do {
201               v' -- option Err (lambda, convert t t');
202               modify (Ast. st (stack := updateStore l (KValue v') (stack st)));
203             }
204           (LStoreloc l, Storage (SValue t')) =>
205             do {
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207               modify (Ast. st (storage := (storage st) (address env := fupd l v' (storage st (address env)))));
208             }
209           (LMemloc l, Memory (MValue t')) =>
210             do {
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217     (KCDptr p, Calldata (MArray x t)) =>
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224               p' -- case sv of Some (KMempr p') => return p' | _ => throw Err;
225               m -- option Err (Ast. cm2m p p' x t cd (memory st));
226               modify (Ast. st (memory := m));
227             }
228           (LStackloc l, Storage _) =>
229             do {
230               sv -- applyf (Ast. accessStore l (stack st));
231               p' -- case sv of Some (KStopr p') => return p' | _ => throw Err;
232               s -- option Err (Ast. cm2s p p' x t cd (storage st (address env)));
233               modify (Ast. st (storage := (storage st) (address env := s)));
234             }
235           (LStoreloc l, _) =>
236             do {
237               s -- option Err (Ast. cm2s p l x t cd (storage st (address env)));
238               modify (Ast. st (storage := (storage st) (address env := s)));
239             }
240           (LMemloc l, _) =>
241             do {
242               m -- option Err (Ast. cm2m p l x t cd (memory st));
243               modify (Ast. st (memory := m));
244             }
245           | _ => throw Err
246         )
247       }
248 )
```



Isabelle/Solidity is a deep embedding of Solidity (v0.5.16) in Isabelle/HOL

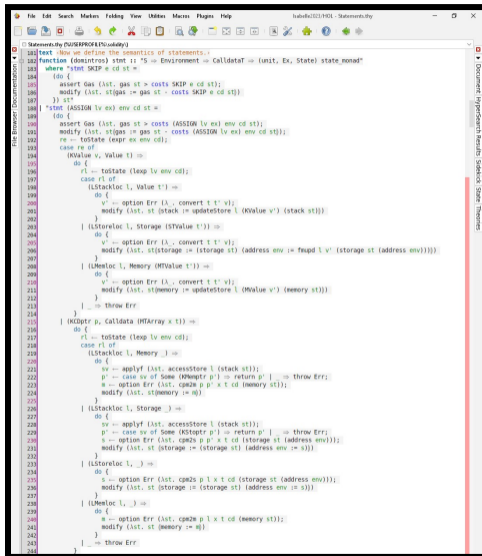
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193     re ==> toState (expr ex env cd);
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195     | (KValue v, Value t) =>
196       do {
197         rl ==> toState (Lexp lv env cd);
198         case rl of
199         | (LStackLoc l, Value t') =>
200           do {
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213           }
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216   ) (KCDptr p, Calldata (MArray x t)) =>
217   do {
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223         p' == case sv of Some (KMemptr p') => return p' | _ => throw Err;
224         m == option Err (Ast.cpm2e p p' x t cd (memory st));
225         modify (Ast.st (memory := m));
226       }
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238       }
239     | (LMemLoc l, _) =>
240       do {
241         m == option Err (Ast.cpm2e p l x t cd (memory st));
242         modify (Ast.st (memory := m));
243       }
244     | _ => throw Err
245   }
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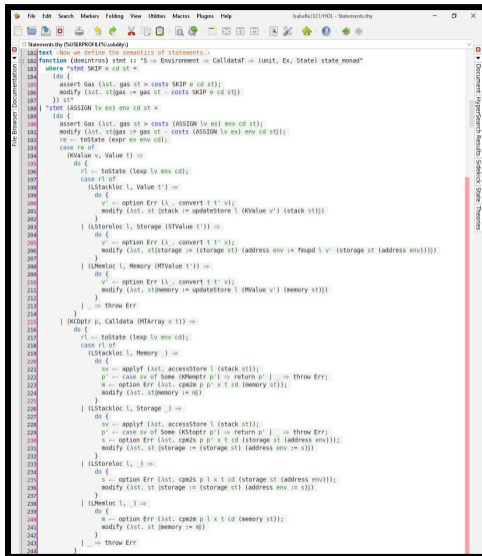
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- *Fallback methods* which are executed with monetary transfers.
- *Different types of stores*, such as storage, memory, calldata, stack.



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191     assert Gas (fst. gas st > costs (ASSIGN lv ex) env cd st);
192     modify (fst. stgas := gas st - costs (ASSIGN lv ex) env cd st);
193     re -> toState (expr ex env cd);
194     case re of
195     | (KInitVal v, Value t) =>
196       do {
197         rl -> toState (Lexp lv env cd);
198         case rl of
199         | (LStackLoc l, Value t') =>
200           do {
201             v' -> option Err (lambda. convert t t' v);
202             modify (fst. st (stack := updateStore l (KValue v') (stack st)));
203           }
204         | (LStoreLoc l, Storage (SValue t')) =>
205           do {
206             v' -> option Err (lambda. convert t t' v);
207             modify (fst. st (storage := (storage st) (address env := fheap l v' (storage st (address env)))));
208           }
209         | (LMemLoc l, Memory (MValue t')) =>
210           do {
211             v' -> option Err (lambda. convert t t' v);
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246     )
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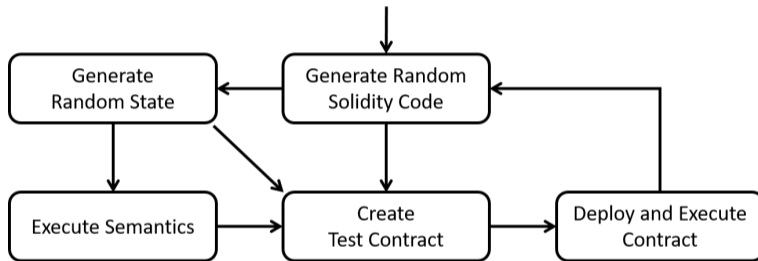
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- *Domain-specific primitives*, such as transfer or balance.
- *Fallback methods* which are executed with monetary transfers.
- *Different types of stores*, such as storage, memory, calldata, stack.
- *Extendable Gas model* to model computational costs.



```
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203           }
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206             v' -- option Err (lambda. convert t' t v);
207             modify (fst. st (storage := (storage st) (address env := fheap l v' (storage st (address env)))));
208           }
209         | (LMemloc l, Memory (MValue t')) =>
210           do {
211             v' -- option Err (lambda. convert t' t v);
212             modify (fst. st (memory := updateStore l (MValue v') (memory st)));
213           }
214         | _ => throw Err
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217       do {
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223             p' -- case sv of Some (KStackptr p') => return p' | _ => throw Err;
224             m -- option Err (fst. cm2s p p' x t cd (memory st));
225             modify (fst. st (memory := m));
226           }
227         | (LStackloc l, Storage _) =>
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246     )
```



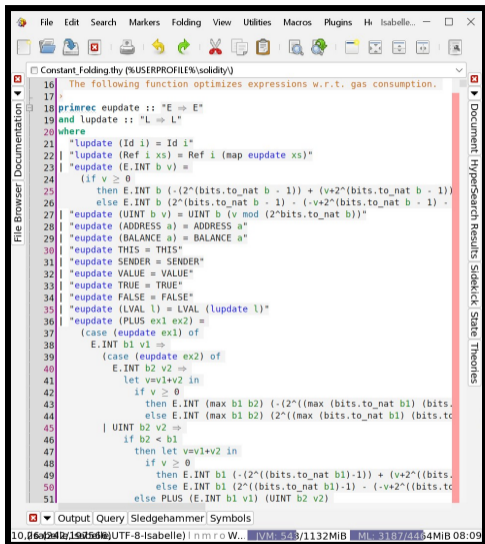
# How to ensure compliance of the semantics



DM and A.D. Brucker.

Conformance Testing of Formal Semantics using Grammar-based Fuzzing. TAP 2022.

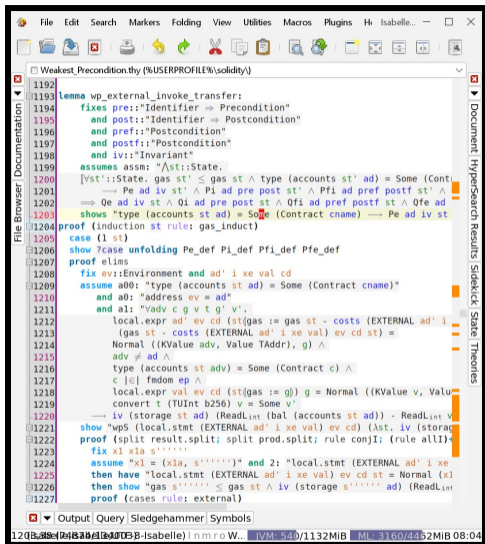
- Verified Constant Solving



```
16 The following function optimizes expressions w.r.t. gas consumption.
17 >
18 primrec eupdate :: "E ⇒ E"
19 and lupdate :: "L ⇒ L"
20 where
21   "lupdate (Id i) = Id i"
22   "lupdate (Ref i xs) = Ref i (map eupdate xs)"
23   "eupdate (E.INT b v) =
24     (if v ≥ 0
25      then E.INT b (-(2bits.to_nat b - 1) + (v+2bits.to_nat b - 1))
26      else E.INT b (2bits.to_nat b - 1 - (-v+2bits.to_nat b - 1)))
27   "eupdate (UINT b v) = UINT b (v mod (2bits.to_nat b))"
28   "eupdate (ADDRESS a) = ADDRESS a"
29   "eupdate (BALANCE a) = BALANCE a"
30   "eupdate THIS = THIS"
31   "eupdate SENDER = SENDER"
32   "eupdate VALUE = VALUE"
33   "eupdate TRUE = TRUE"
34   "eupdate FALSE = FALSE"
35   "eupdate (LVAL l) = LVAL (lupdate l)"
36   "eupdate (PLUS ex1 ex2) =
37     (case (eupdate ex1) of
38      E.INT b1 v1 ⇒
39        (case (eupdate ex2) of
40         E.INT b2 v2 ⇒
41           let v=v1+v2 in
42             if v ≥ 0
43               then E.INT (max b1 b2) (-(2((max (bits.to_nat b1) (bits.to_nat b2) - 1))) + (v+2((max (bits.to_nat b1) (bits.to_nat b2) - 1))))
44             else E.INT (max b1 b2) (2((max (bits.to_nat b1) (bits.to_nat b2) - 1)) - (-v+2((max (bits.to_nat b1) (bits.to_nat b2) - 1))))
45         | UINT b2 v2 ⇒
46           if b2 < b1
47             then let v=v1+v2 in
48                  if v ≥ 0
49                    then E.INT b1 (-(2((bits.to_nat b1) - 1))) + (v+2((bits.to_nat b1) - 1))
50                    else E.INT b1 (2((bits.to_nat b1) - 1)) - (-v+2((bits.to_nat b1) - 1))
51             else PLUS (E.INT b1 v1) (UINT b2 v2)
```

# Applications

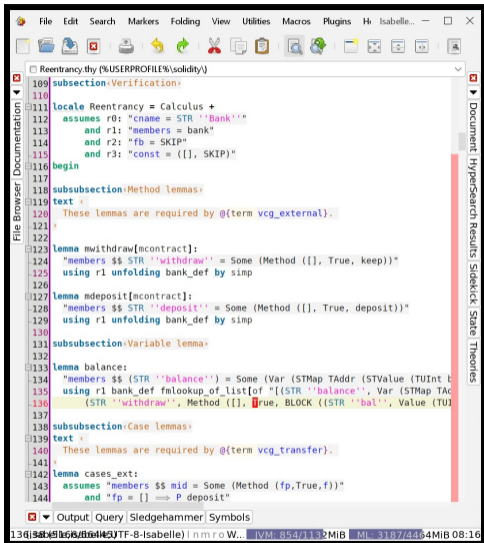
- Verified Constant Solving
- Soundness of SSCalc



```
1192 lemma wp_external_invoke_transfer:
1193   fixes pre::"Identifier ⇒ Precondition"
1194   and post::"Identifier ⇒ Postcondition"
1195   and pref::"Postcondition"
1196   and postf::"Postcondition"
1197   and iv::"Invariant"
1198   and assm: "\st::State.
1199     [\vst::State. gas st' ≤ gas st ∧ type (accounts st ad) = Some (Contract cname)
1200     → Pe ad iv st' ∧ Pi ad pre post st' ∧ Pfi ad pref postf st' ∧
1201     ⇒ Qe ad iv st ∧ Qi ad pre post st ∧ Qfi ad pref postf st ∧ Qfe ad
1202     shows "type (accounts st ad) = Some (Contract cname) → Pe ad iv st
1203 proof (induction st rule: gas_induct)
1204 case (l st)
1205 show ?case unfolding Pe_def Pi_def Pfi_def Pfe_def
1206 proof elim
1207   fix ev::Environment and ad' i xe val cd
1208   assume a00: "type (accounts st ad) = Some (Contract cname)"
1209   and a0: "address ev = ad"
1210   and a1: "\adv c g v t g' v'.
1211     local.expr ad' ev cd (st(gas := gas st - costs (EXTERNAL ad' i xe val) ev cd st) =
1212     Normal ((KValue adv, Value TAddr), g) ∧
1213     adv ≠ ad ∧
1214     type (accounts st adv) = Some (Contract c) ∧
1215     c |∈| fmdom ep ∧
1216     local.expr val ev cd (st(gas := g)) g = Normal ((KValue v, Value TAddr), t) ∧
1217     convert t (TUInt b256) v = Some v"
1218   iv (storage st ad) (ReadLInt (bal (accounts st ad)) - ReadLInt v)
1219   show "wpS (local.stmt (EXTERNAL ad' i xe val) ev cd) (\st. iv (storage st ad) (ReadLInt (bal (accounts st ad)) - ReadLInt v))"
1220 proof (split result.split; split prod.split; rule conjI)
1221   fix x1 x1a s''''''
1222   assume "x1 = (x1a, s'''''')" and 2: "local.stmt (EXTERNAL ad' i xe val) ev cd st = Normal (x1)"
1223   then have "local.stmt (EXTERNAL ad' i xe val) ev cd st = Normal (x1)"
1224   then show "gas s'''''' ≤ gas st ∧ iv (storage s'''''' ad) (ReadLInt v)"
1225   proof (cases rule: external)
```

# Applications

- Verified Constant Solving
- Soundness of SSCalc
- Verified Banking



```
109 subsection<Verification>
110
111 locale Reentrancy = Calculus +
112   assumes r0: "cname = STR 'Bank'"
113   and r1: "members = bank"
114   and r2: "fb = SKIP"
115   and r3: "const = ([], SKIP)"
116 begin
117
118 subsection<Method lemmas>
119 text <
120   These lemmas are required by @(term vcg_external).
121 >
122
123 lemma mwithdraw[mcontract]:
124   "members $$ STR 'withdraw' = Some (Method ([], True, keep))"
125   using r1 unfolding bank_def by simp
126
127 lemma mdeposit[mcontract]:
128   "members $$ STR 'deposit' = Some (Method ([], True, deposit))"
129   using r1 unfolding bank_def by simp
130
131 subsection<Variable lemma>
132
133 lemma balance:
134   "members $$ (STR 'balance') = Some (Var (STMap TAddr (STValue (TUnit b
135   using r1 bank_def fmlookup_of_List[of "{(STR 'balance', Var (STMap TAc
136   (STR 'withdraw', Method ([], True, BLOCK ((STR 'bal', Value (TU
137
138 subsection<Case lemmas>
139 text <
140   These lemmas are required by @(term vcg_transfer).
141 >
142 lemma cases_ext:
143   assumes "members $$ mid = Some (Method (fp,True,f))"
144   and "fp = [] ==> P deposit"
```





What is achieved so far

- Formalisation of a subset of Solidity in Isabelle/HOL
  - *Conservative extension* guarantees semantic consistency
  - *Deep embedding* allows to reason about the language itself



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- Formalisation of a subset of Solidity in Isabelle/HOL
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- Used in several case studies to verify ...
  - Gas-optimizer
  - soundness of Solidity calculus
  - concrete Solidity contracts



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- Formalisation of a subset of Solidity in Isabelle/HOL
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What are we currently working on

- *Shallow embedding* to improve automation for the verification of contracts






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- Formalisation of a subset of Solidity in Isabelle/HOL
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- Used in several case studies to verify ...
  - Gas-optimizer
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What are we currently working on

- *Shallow embedding* to improve automation for the verification of contracts
- First results are promising!

## References I

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## 4 Language Features

- Fixed-size Integer Types
- Domain-specific Primitives
- Gas Model
- Method Calls
- Complex Data Types
- Assignments with Different Semantics

## 5 Testing

## 6 Example Applications

- Verified Constant Solving
- SSCalc
- Banking Contract

# Fixed-size Integer Types

- Signed and unsigned integers from 8...256 bits (with steps of 8 bits)
- Signed integer types are only compatible with unsigned types of smaller size
- If a value is too large for a size a silent overflow will occur



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assert(int8(200) == int8(-56));
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Solidity

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assert(int8(200) == int8(-56)); //true  
  
assert(uint8(200) == uint8(-56)); //true  
  
assert(uint8(200) + int16(32600) == int16(-32736));
```

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assert(uint16(100) + int16(32700));
```

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assert(uint8(200) + int16(32600) == int16(-32736)); //true  
  
assert(uint16(100) + int16(32700)); //compiler error
```

Solidity



# Domain-specific Primitives

- External vs. contract accounts
- Query account balances
- Transfer money



## Language Features

Fixed-size Integer Types

Domain-specific Primitives

Gas Model

Method Calls

Complex Data Types

Assignments with Different  
Semantics

## Testing

### Example Applications

Verified Constant Solving

SSCalc

Banking Contract

## Domain-specific Primitives

- External vs. contract accounts
- Query account balances
- Transfer money

```
uint256 x = 0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.balance;  
uint256 y = address(this).balance;
```

Solidity

## Domain-specific Primitives

- External vs. contract accounts
- Query account balances
- Transfer money

```
uint256 x = 0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.balance;  
uint256 y = address(this).balance;
```

```
0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.transfer(1000);
```

Solidity

## Domain-specific Primitives

- External vs. contract accounts
- Query account balances
- Transfer money

```
uint256 x = 0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.balance;  
uint256 y = address(this).balance;  
  
0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.transfer(1000);  
  
assert(0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.balance == x+1000);  
//true
```

Solidity

## Domain-specific Primitives

- External vs. contract accounts
- Query account balances
- Transfer money

```
uint256 x = 0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.balance;  
uint256 y = address(this).balance;  
  
0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.transfer(1000);  
  
assert(0xAb8483F64d9C6d1EcF9b849Ae677dD3315835cb2.balance == x+1000);  
//true  
  
assert(address(this).balance == y-1000); //true
```

Solidity

# Gas Model

- Execution costs Gas
- Programs are guaranteed to terminate
- No specification for Gas costs at Solidity level

# Gas Model

- Execution costs Gas
- Programs are guaranteed to terminate
- No specification for Gas costs at Solidity level

```
while (true) {}  
//terminates with an out of gas exception
```

Solidity

## Method Calls

Recently we added support for *method calls*

- Internal vs. external
- Send money with external calls
- Money transfer triggers fallback

### Language Features

Fixed-size Integer Types

Domain-specific Primitives

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## Method Calls

Recently we added support for *method calls*

- Internal vs. external
- Send money with external calls
- Money transfer triggers fallback

### Solidity

```
contract R {  
    mapping(address => uint256) map;  
  
    function rcv() external payable {  
        map[msg.sender] = msg.value;  
    }  
}
```

### Solidity

```
contract S {  
    R rec;  
  
    constructor(R r) public payable {  
        rec = r;  
    }  
  
    function snd(uint256 v) public {  
        rec.rcv.value(v)();  
    }  
}
```

## Complex Data Types

- Three types of stores: storage, memory, calldata
- Mappings can only be kept in storage
- Arrays can be kept in all types of stores

### Language Features

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## Complex Data Types

- Three types of stores: storage, memory, calldata
- Mappings can only be kept in storage
- Arrays can be kept in all types of stores

### Solidity

```
contract Example {
    mapping(address => uint256) myMapping; //storage map

    uint8[2][3] myStorageArray; //storage array

    //calldata array
    function example(uint8[2] calldata myCDArray) external {
        uint8[2] storage myPointer = myStorageArray[1]; //storage pointer

        uint8[2] memory myMemoryArray; //memory array
    }
}
```

## Assignments with Different Semantics

- Assignment between memory moves pointer
- Assignment between storage copies (except for pointers)
- Assignment between memory and storage copies

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- Assignment between memory moves pointer
- Assignment between storage copies (except for pointers)
- Assignment between memory and storage copies

Solidity

```
//initialized with 0
int [2] memory x;
int [2] memory y;

x=y;
x[1]=1;

assert(y[1] == 1); //true
```

Solidity

```
int [2] [2] memory x;
int [2] [2] memory y;

x[1]=y[1];
x[0][0]=1;
x[1][1]=1;

assert(y[0][0] == 1); //false
assert(y[1][1] == 1); //true
```

## Assignments with Different Semantics

- Assignment between memory moves pointer
- Assignment between storage copies (except for pointers)
- Assignment between memory and storage copies

Solidity

```
contract Example {
  //initialized with 0
  int[2] storage y;

  function example() public {
    int[2] storage x=y;

    x[1]=1;

    assert(y[1]==1); //true
  }
}
```

Solidity

```
contract Example {
  //initialized with 0
  int[2] storage x;
  int[2] storage y;

  function example() public {
    x = y;
    x[1]=1;

    assert(y[1]==1); //false
  }
}
```

## Assignments with Different Semantics

- Assignment between memory moves pointer
- Assignment between storage copies (except for pointers)
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Solidity

```
contract Example {
  //initialized with 0
  int[2] storage y;

  function example() public {
    int[2] memory x = y;

    x[1]=1;

    assert(y[1] == 1); //false
  }
}
```

Solidity

```
contract Example {
  //initialized with 0
  int[2] storage y;

  function example() public {
    int[2] memory x = y;
    x[1]=y[1];
    x[1][1]=1;

    assert(y[1][1]==1); //false
  }
}
```

# Example

```
contract TestContract0 {
  uint8 v_u8_s8;
  mapping(uint16 => uint8) v_m_u16_u8_9;
  bool [1][2] a_b_12_s5;
  ...
  function test () public {
    uint104 v_u104_m2;
    uint104 [1][1] memory a_u104_11_m2;
    ...
    v_u104_m2=14622709355569675963178665339646;
    v_m_u16_u8_9[59381]=79;
    ...
    int8 counter1=int8(0);
    while((v_m_u224_s240_1[uint224(444)]==(v_u216_s1-v_u104_m2)) && counter1<int8(10)){
      0xf7218C33533a3F22e3296F8b1DC0074B399355Eb.transfer(v_m_u16_u8_9[uint16(0)]);
      counter1=counter1+int8(1);
    }
    ...
    Assert.equal(v_m_u16_u8_9[59381]==79, true);
    Assert.equal(a_u104_11_m2[0][0]==8130097819054169632795960896007, true);
    Assert.equal(0xf7218C33533a3F22e3296F8b1DC0074B399355Eb
      .balance==10000000000000000000, true);
    ...
  }
}
```

Extracted storage variables

Extracted memory/stack variables

Generated input state

Generated program

Computed result state

## Language Features

- Fixed-size Integer Types
- Domain-specific Primitives
- Gas Model
- Method Calls
- Complex Data Types
- Assignments with Different Semantics

## Testing

### Example Applications

- Verified Constant Solving
- SSCalc
- Banking Contract



# Verified Constant Solving

```
int16 x;
```

```
// costs 20 Gas
```

```
x = int16(250) + uint8(500);
```

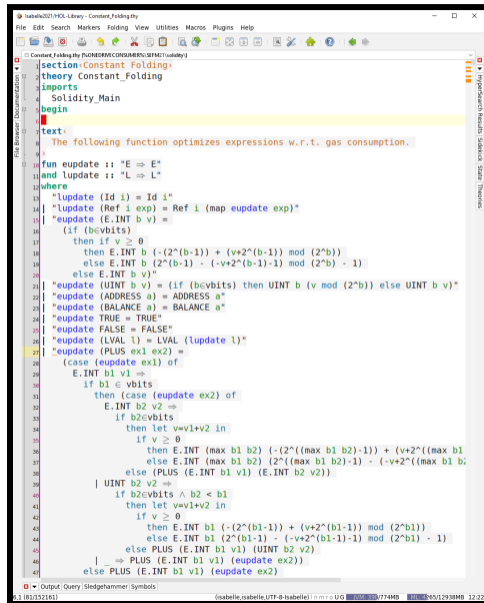
Solidity

```
int16 x;
```

```
// costs 8 Gas
```

```
x = int16(494);
```

Solidity



```
Isabelle2021-1405 - Library - Constant_Folding.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Constant_Folding.thy (FUNCTIONS CONSUMERS: SEMANTICS Isabelle)
1 section Constant_Folding
2 theory Constant_Folding
3 imports
4   Solidity_Main
5 begin
6
7 text
8   The following function optimizes expressions w.r.t. gas consumption.
9
10 fun eupdate :: "E => E"
11 and lupdate :: "L => L"
12 where
13   "lupdate (Id i) = Id i"
14 | "lupdate (Ref i exp) = Ref i (map eupdate exp)"
15 | "eupdate (E.INT b v) =
16   (if (b<evbits)
17    then if v >= 0
18         then E.INT b (-(2^(b-1)) + (v+2^(b-1)) mod (2^b))
19         else E.INT b (2^(b-1) - (-v+2^(b-1)-1) mod (2^b) - 1)
20    else E.INT b v)"
21 | "eupdate (UINT b v) = (if (b<evbits) then UINT b (v mod (2^b)) else UINT b v)"
22 | "eupdate (ADDRESS a) = ADDRESS a"
23 | "eupdate (BALANCE a) = BALANCE a"
24 | "eupdate TRUE = TRUE"
25 | "eupdate FALSE = FALSE"
26 | "eupdate (LVAL l) = LVAL (lupdate l)"
27 | "eupdate (PLUS ex1 ex2) =
28   (case (eupdate ex1) of
29    E.INT b1 v1 =>
30      if b1 <= vbits
31        then (case (eupdate ex2) of
32         E.INT b2 v2 =>
33           if b2<evbits
34             then let v=v1+v2 in
35                  if v >= 0
36                    then E.INT (max b1 b2) (-(2^((max b1 b2)-1)) + (v+2^((max b1
37                    else E.INT (max b1 b2) (2^((max b1 b2)-1) - (-v+2^((max b1 b
38         else (PLUS (E.INT b1 v1) (E.INT b2 v2))
39    | UINT b2 v2 =>
40      if b2<evbits & b2 < b1
41        then let v=v1+v2 in
42             if v >= 0
43               then E.INT b1 (-(2^(b1-1)) + (v+2^(b1-1)) mod (2^b1))
44               else E.INT b1 (2^(b1-1) - (-v+2^(b1-1)-1) mod (2^b1) - 1)
45         else PLUS (E.INT b1 v1) (UINT b2 v2)
46    | _ => PLUS (E.INT b1 v1) (eupdate ex2))
47   else PLUS (E.INT b1 v1) (eupdate ex2))
48
49 Output Query Sledgehammer Symbols
1.1 (81/132161) (isabelle.isabelle.UTF-8-isabelle) © 2021 UG 000000077448B 000000050129388B 12:22
```



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#### Verified Constant Solving

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## Specification

- Invariant over member variables and balance
- Pre/post-conditions for internal methods

## Verification

- Constructor establishes invariant
- External methods preserve invariant
- Preconditions imply postconditions for internal methods

Solidity

```
contract Example {
  uint x;
  constructor(uint y, ...) public {
    ... x = y; ...
  }
  function int1(uint y, ...) internal {
    ... ad1.call.value(1 ether)(abi.
      encodeWithSignature("ext()")); ...
  }
  function ext() external {
    ... int1(5, ...); ...
    ... ad2.transfer(1 ether); ...
  }
  function () external payable {
    ...
  }
}
```

Diego Marmosler



## Language Features

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Example  
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Verified Constant Solving

SSCalc

Banking Contract

# Verification of Banking Contract

$$\sum_a \text{balances}(a) \leq \text{balance}$$

```
contract Bank {
  mapping(address => uint256) balances;

  function deposit() public payable {
    balances[msg.sender] = balances[msg.sender] + msg.value;
  }

  function withdraw() public {
    uint256 bal = balances[msg.sender];
    balances[msg.sender] = 0;
    msg.sender.transfer(bal);
  }
}
```

Solidity

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Banking Contract