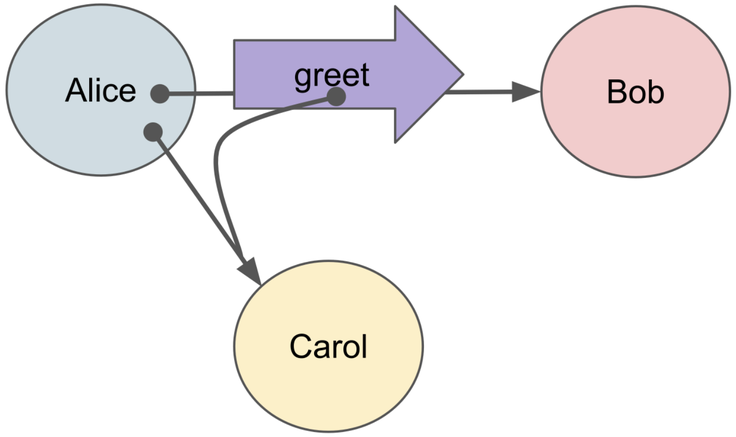
VeTSS 2024

Reasoning with Object Capabilities

Sophia Drossopoulou, Imperial College London



James Noble

Susan Eisenbach

Julian Mackay

need new encapsulation features

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Ownership types?

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Ownership types?

… WHAT should these features guarantee?

Background,
Problem,
OCAP,
Our remit

The Background:

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B1: Internal (trusted) and External (untrusted) objects are intertwined.

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The Problem: Mitigate the arising uncertainty.

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Not our remit: Forbid external access to capabilty.

Background,
Problem,
OCAP,
Our remit

— in a diagram —

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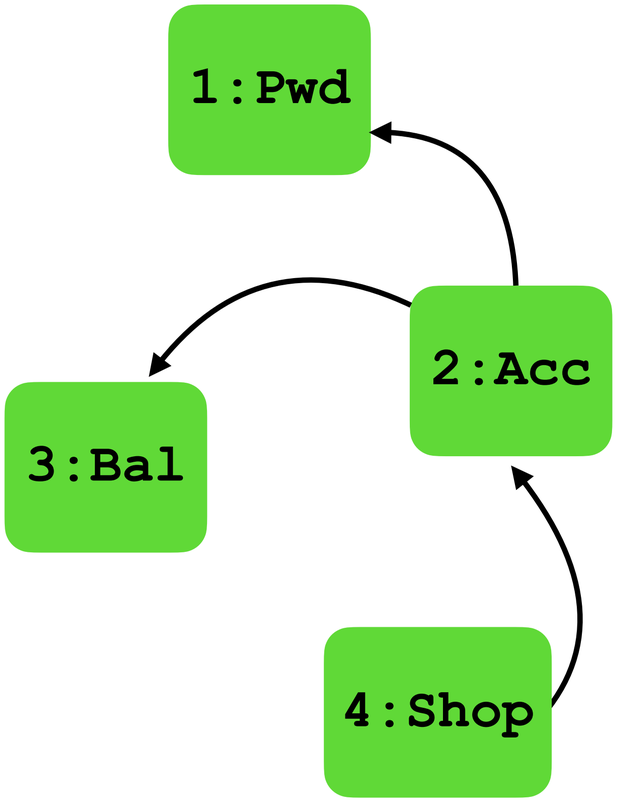

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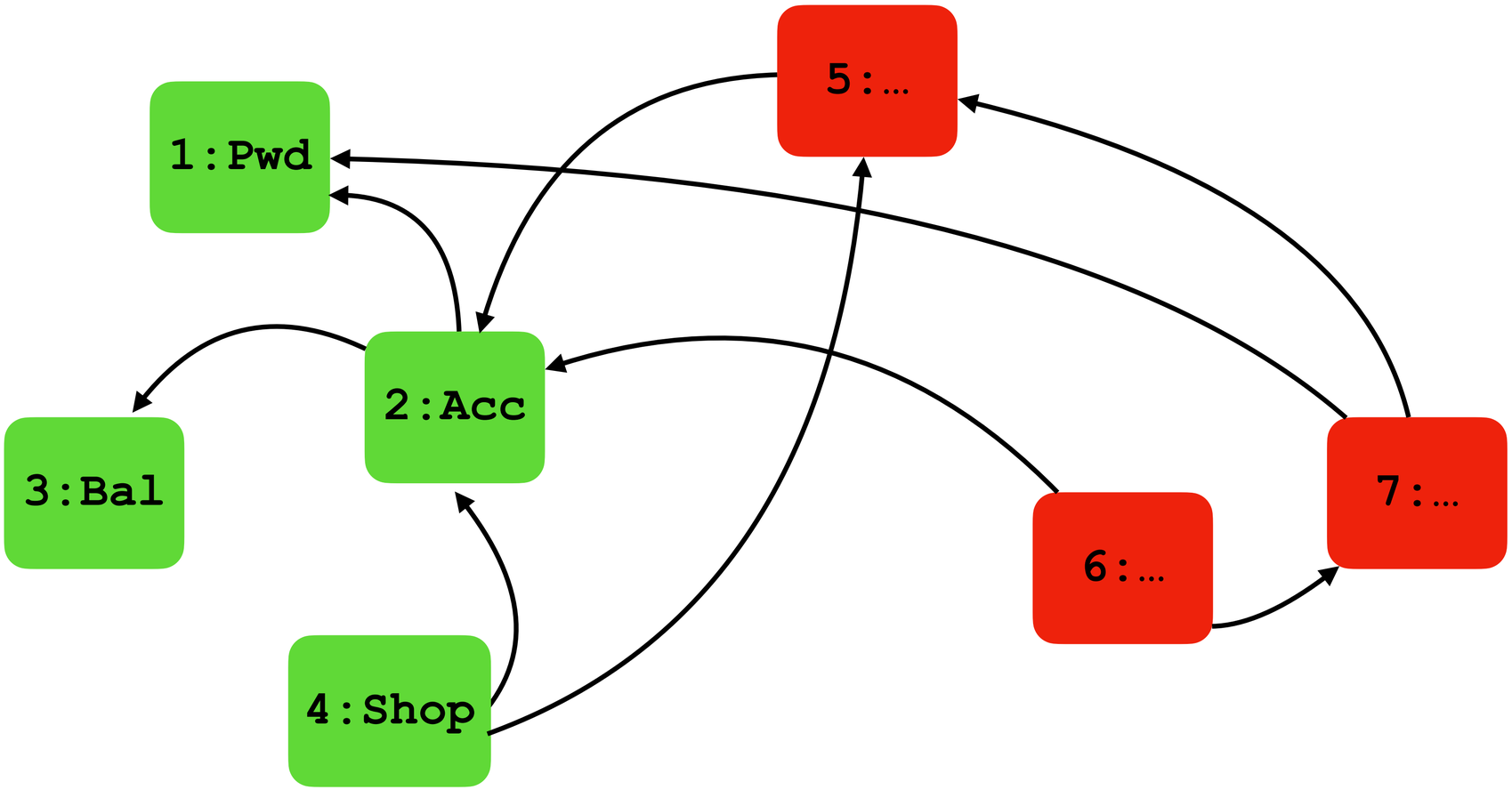
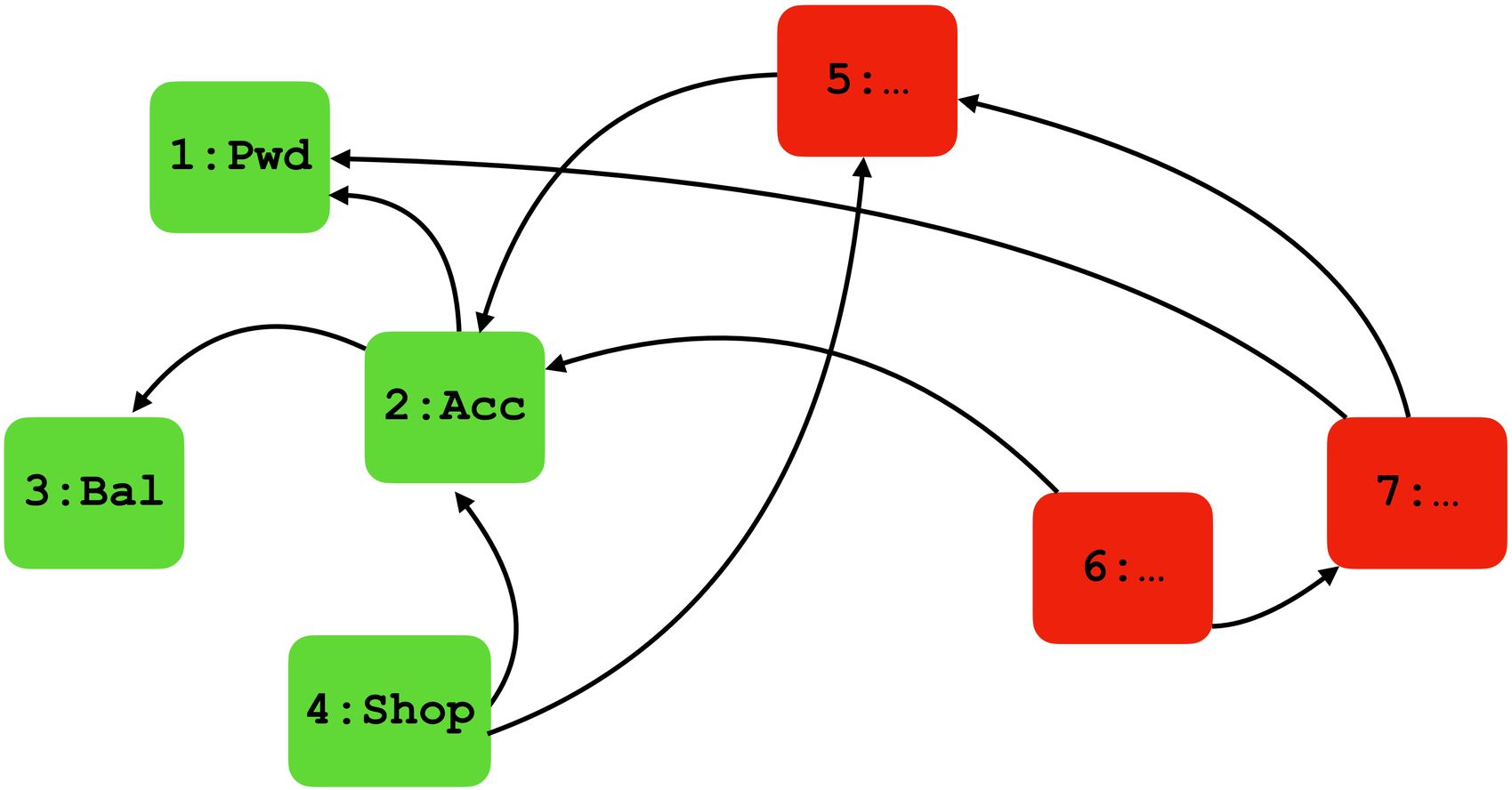


The Background:

B2: External objects may call methods on internal objects.



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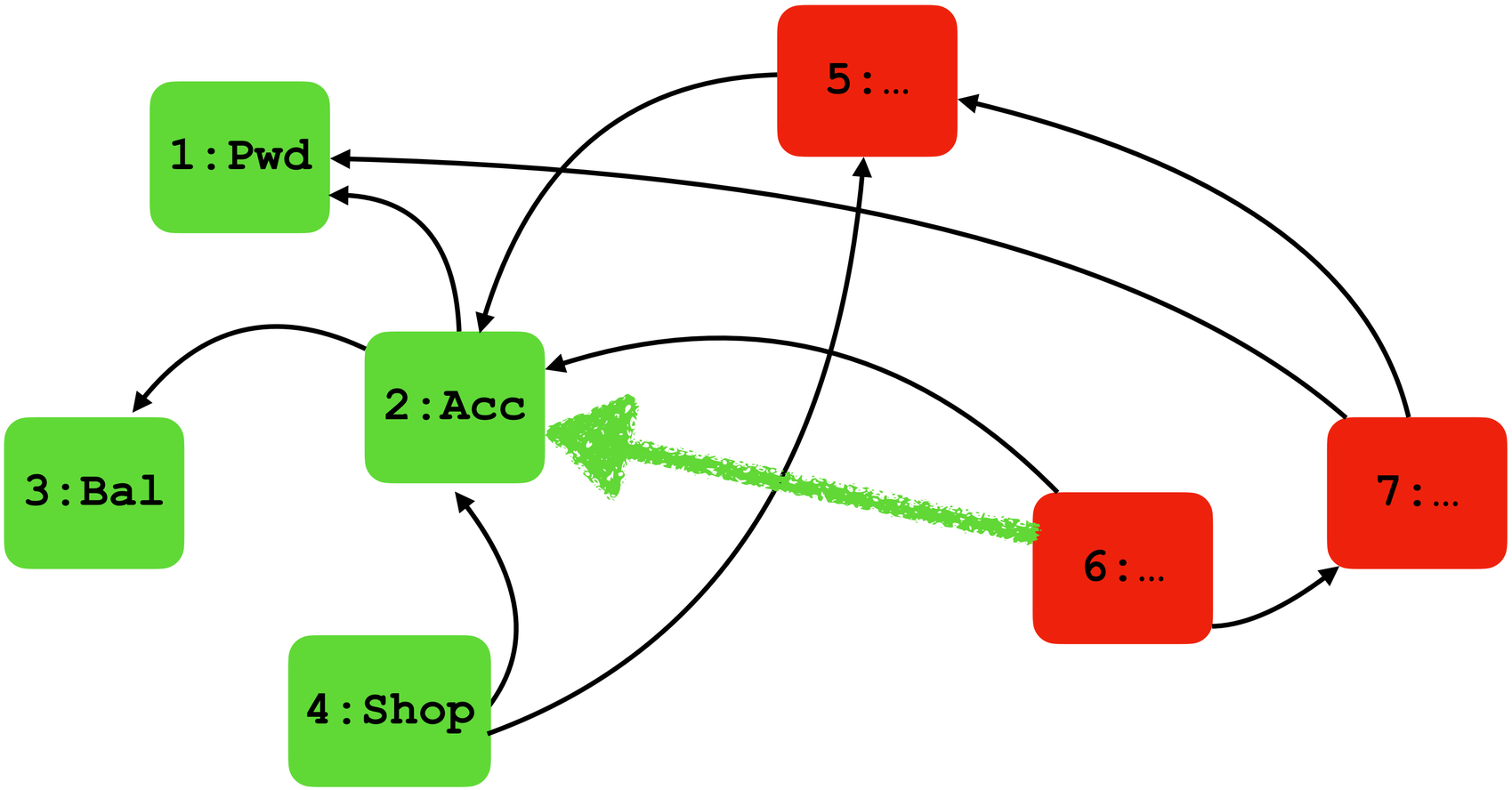

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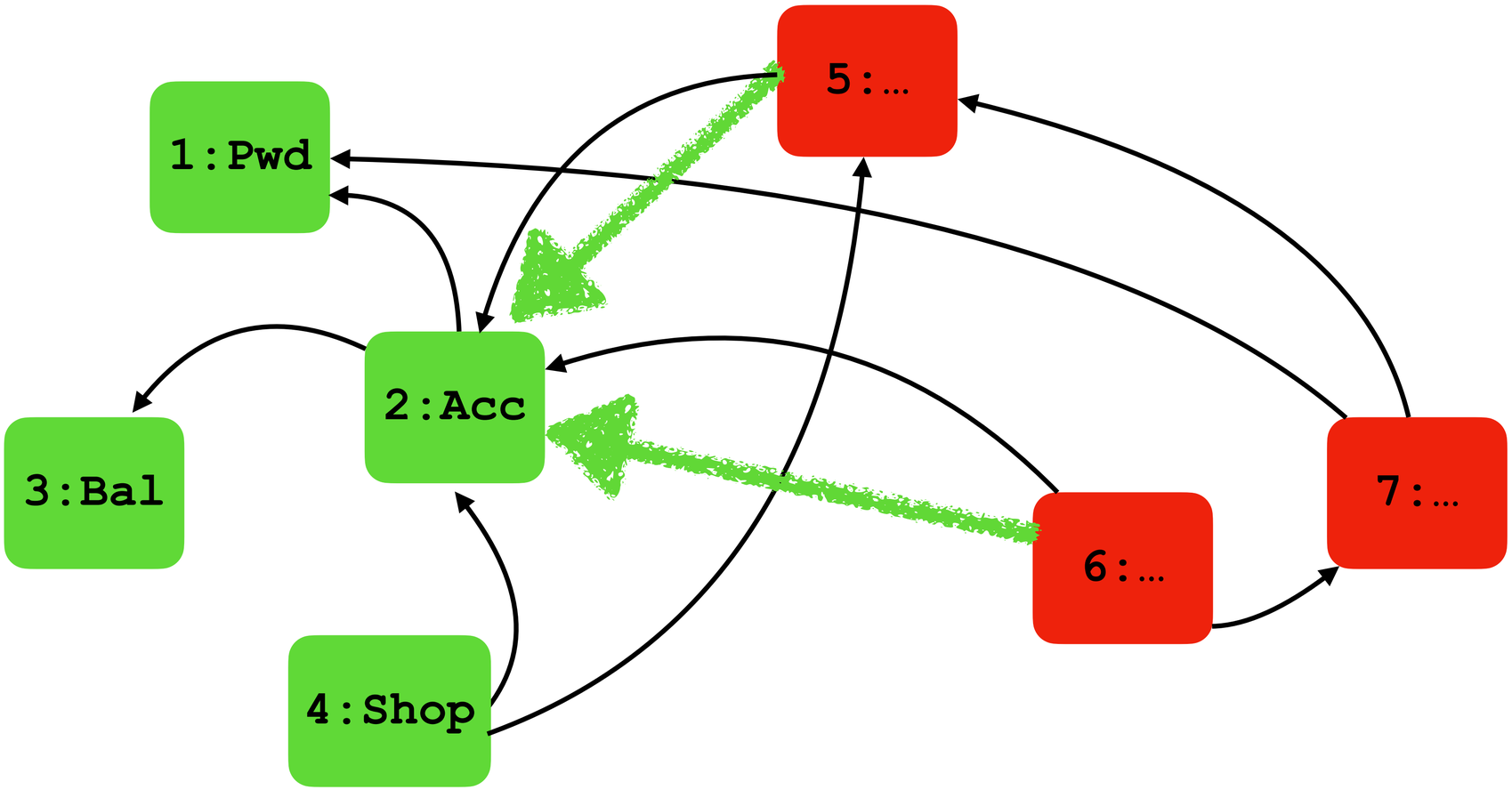

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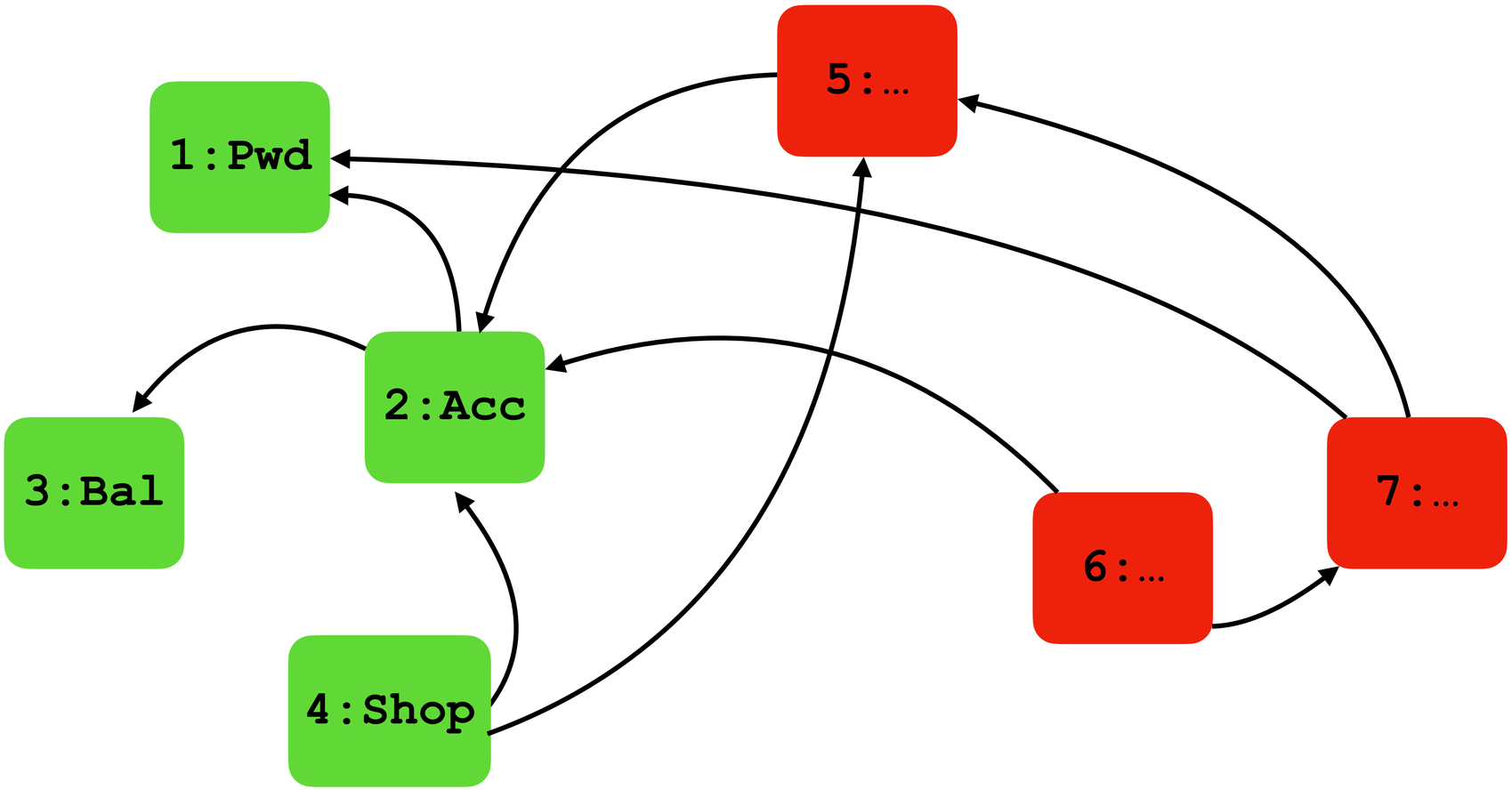

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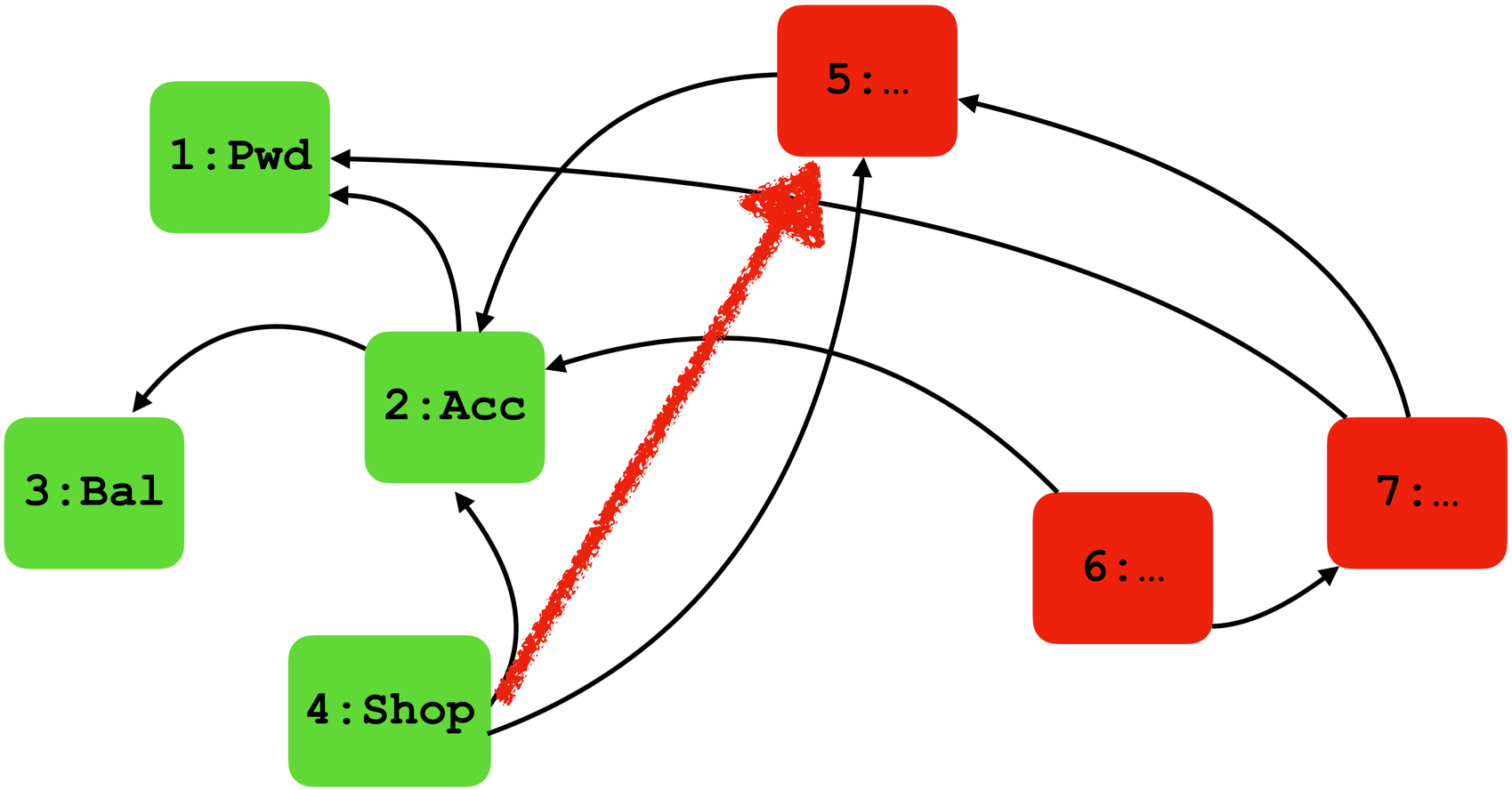
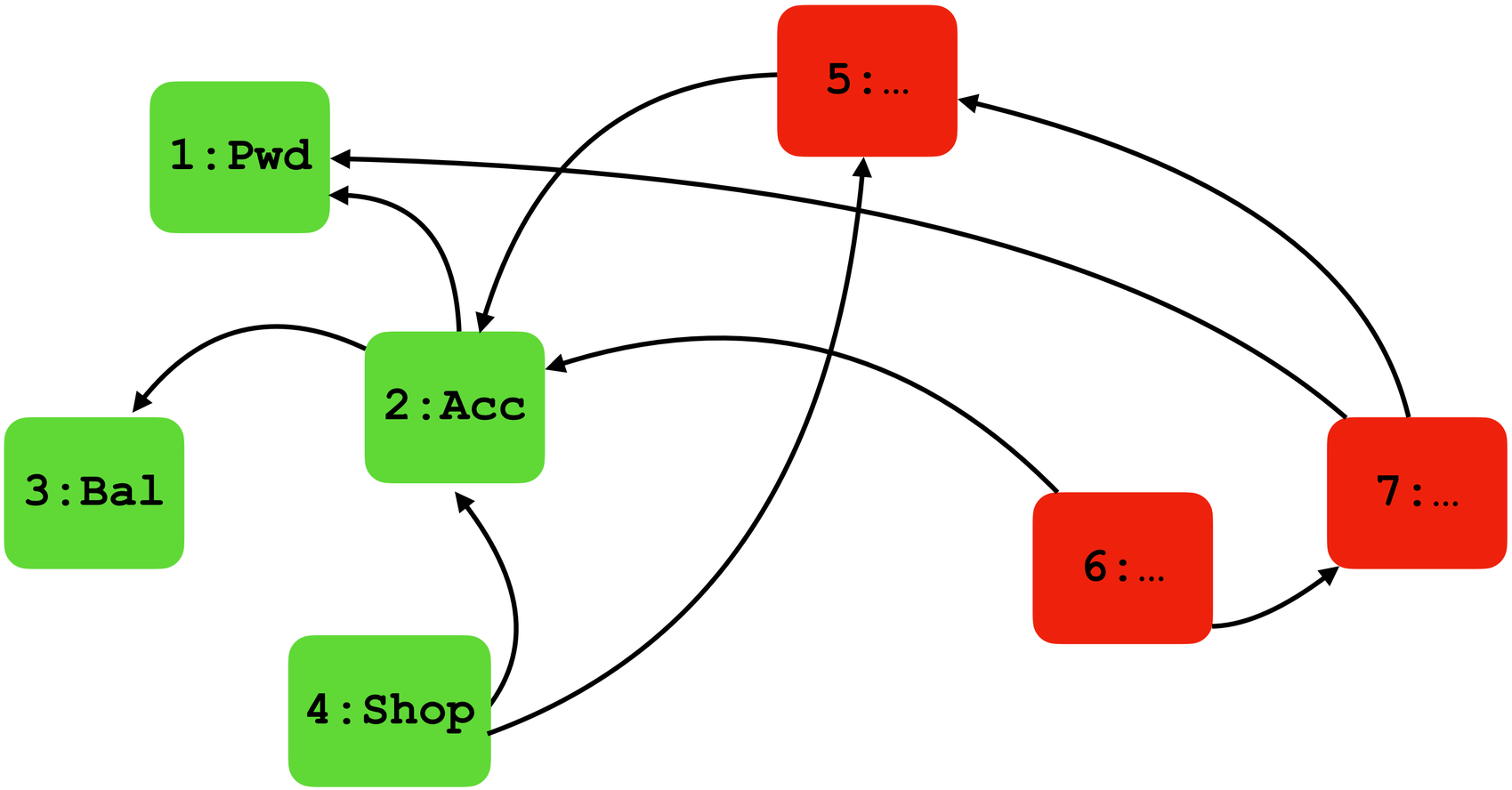
B3: Internal objects may call methods on external objects.



The Problem:

OCAP Solution:



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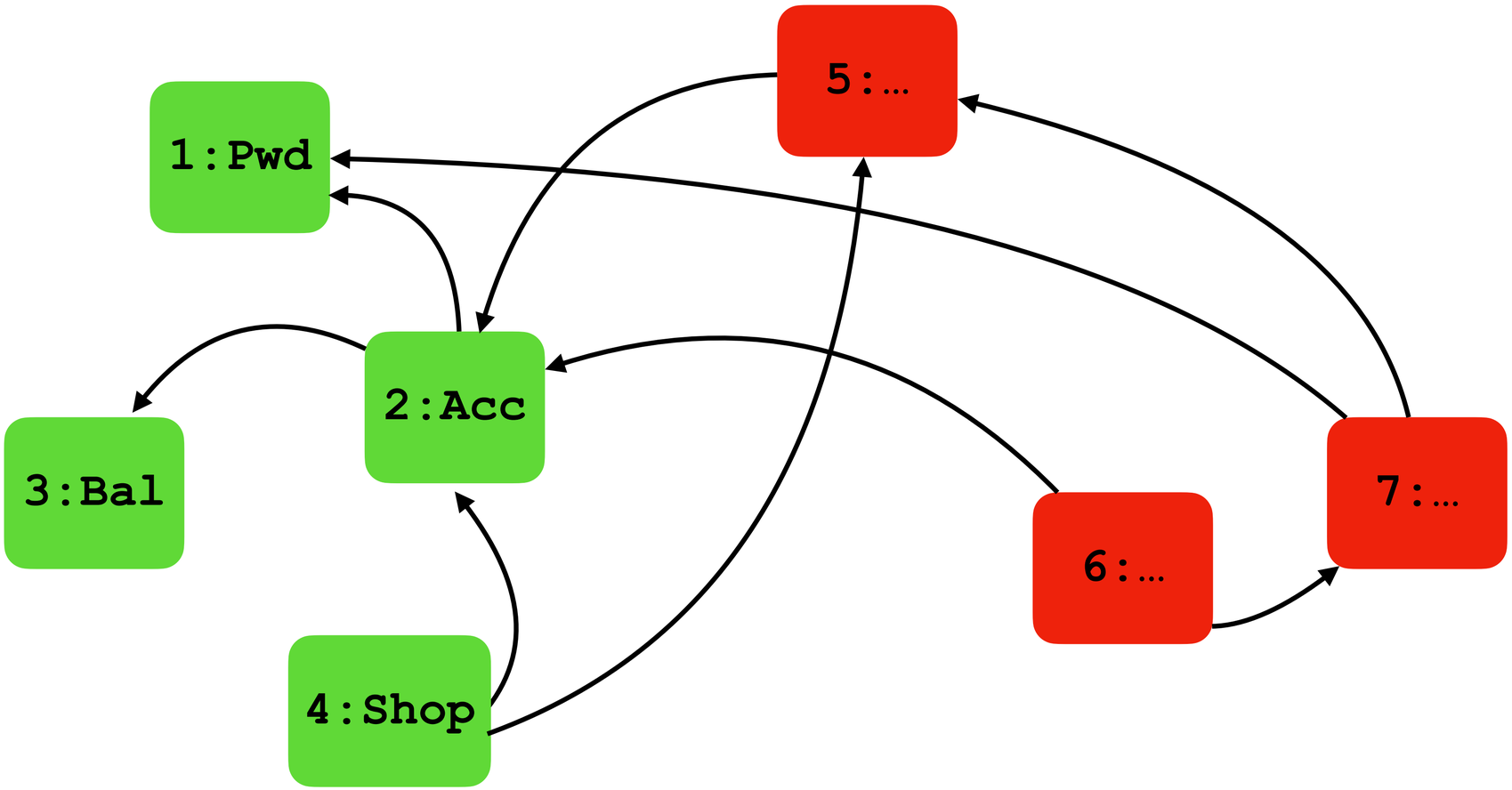
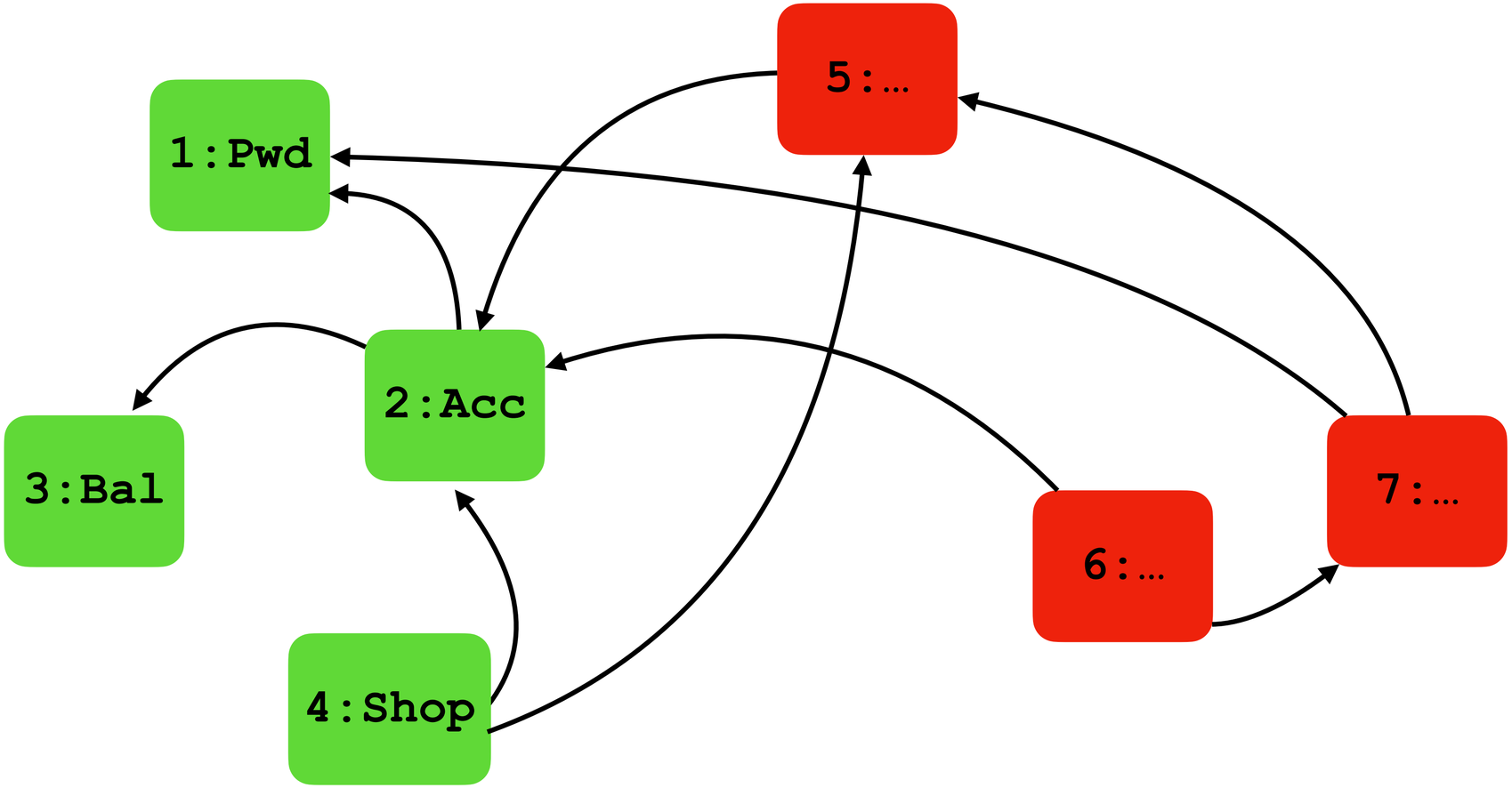


The

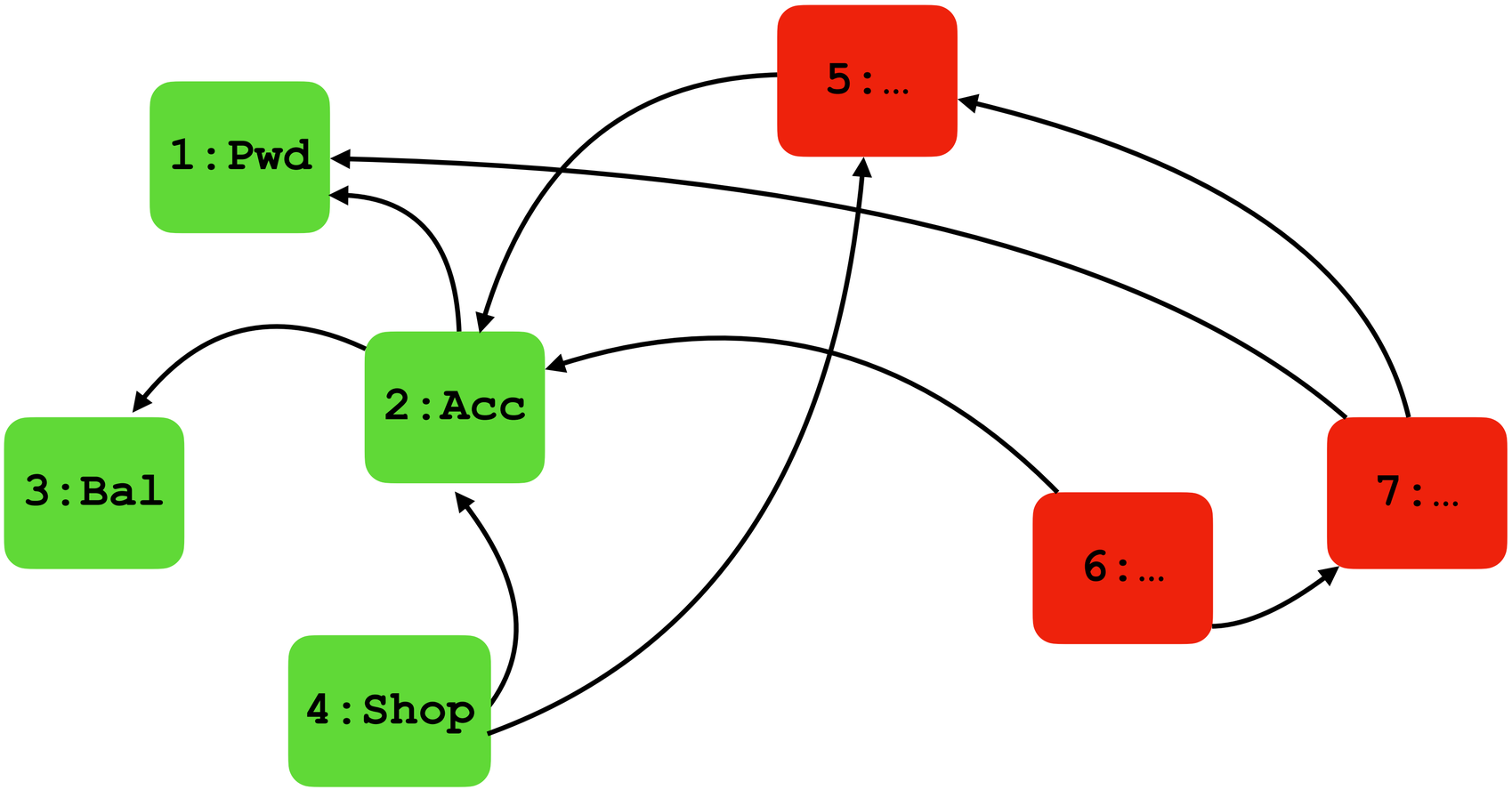
OCA

Capability as *guard* for a certain effect.



Our remit:

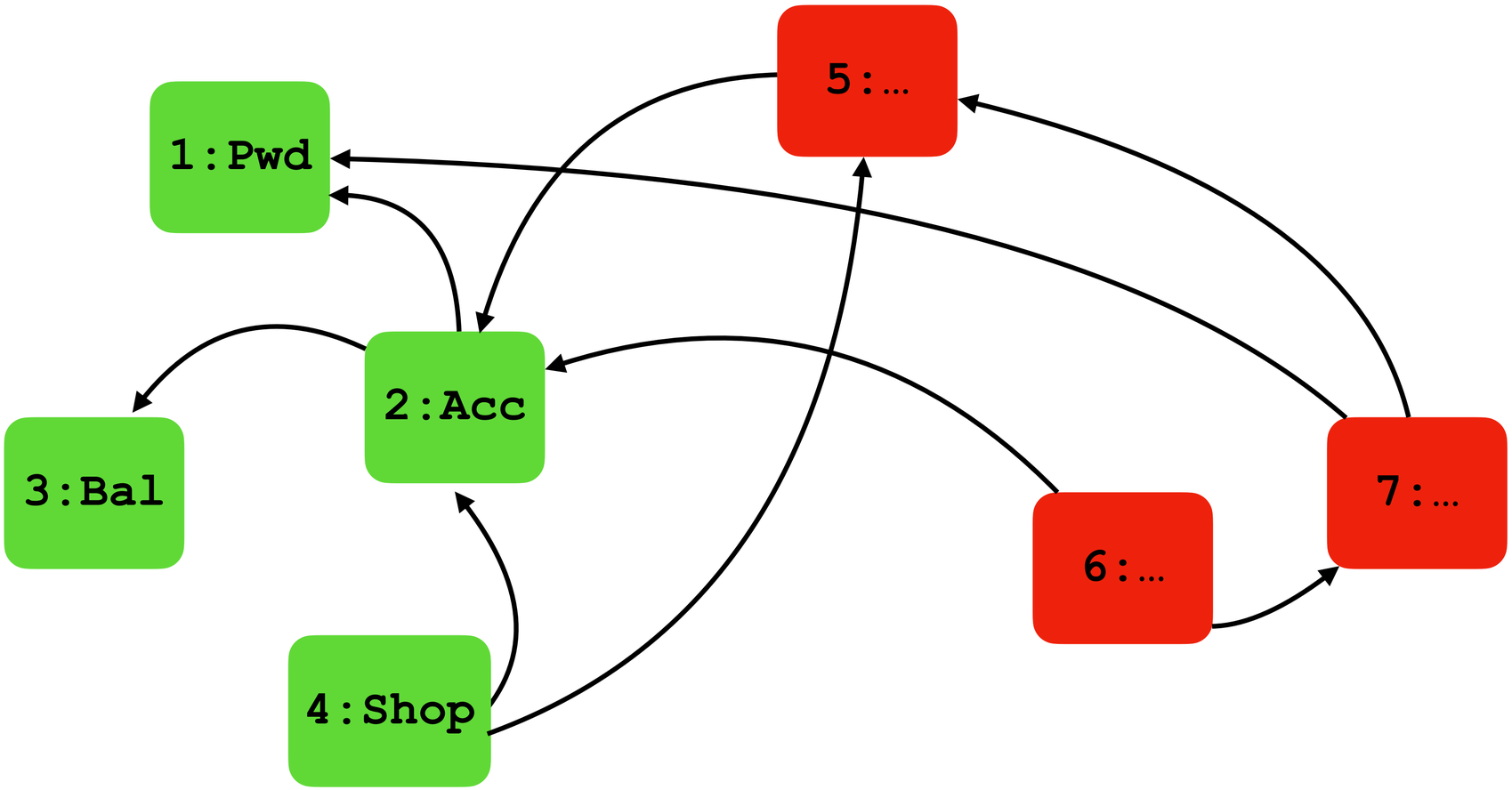
R1: Specify that eventlual. external access to capability necessary for effect.



Our remit:

R1: Specify that eventlual. external access to capability necessary for effect.

S1: without eventl. access to account no change in balance

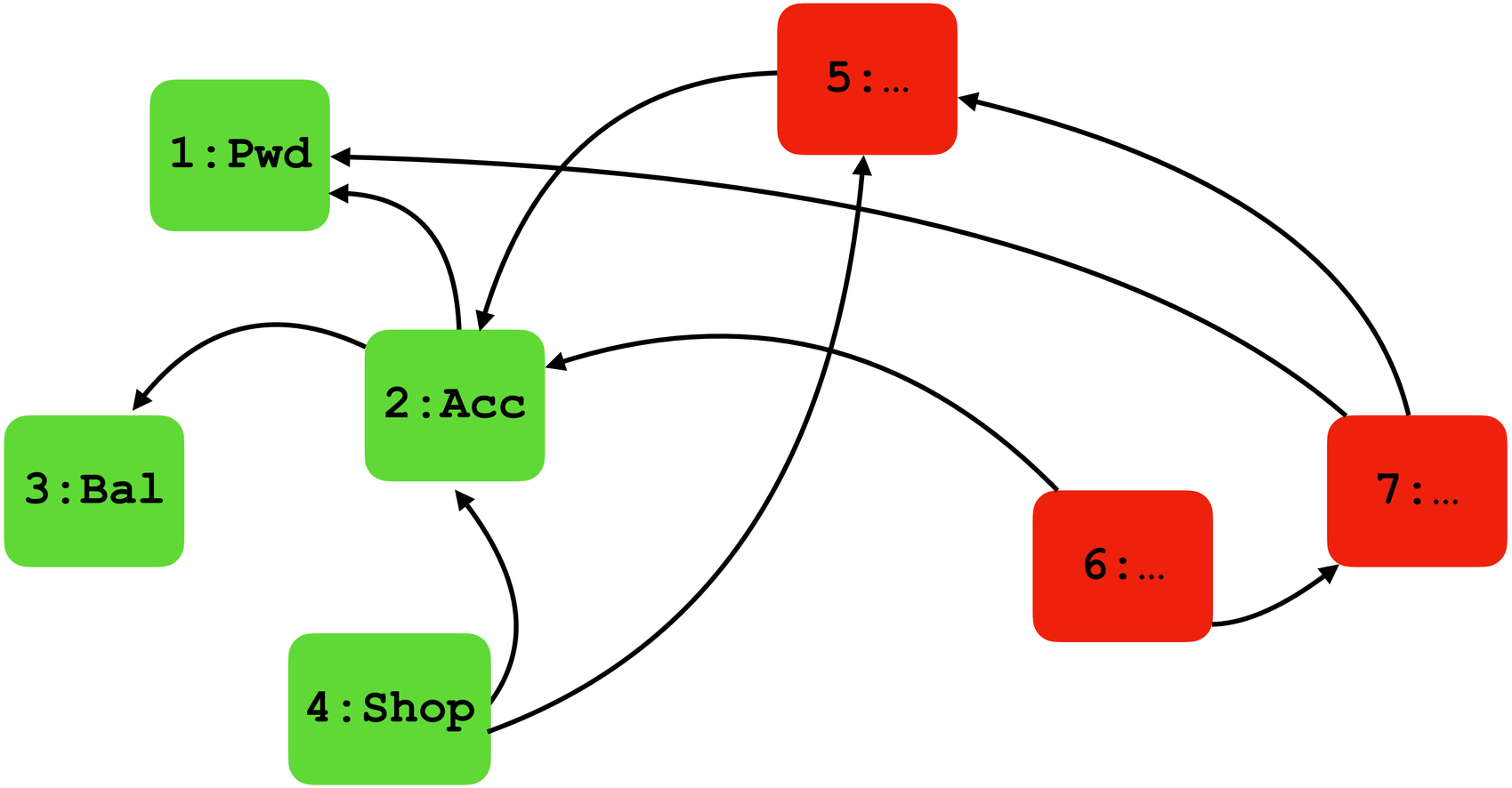


Our remit:

R1: Specify that eventlual. external access to capability necessary for effect.

S1: without eventl. access to account no change in balance

S2: without eventl. access to password no decrease in balance



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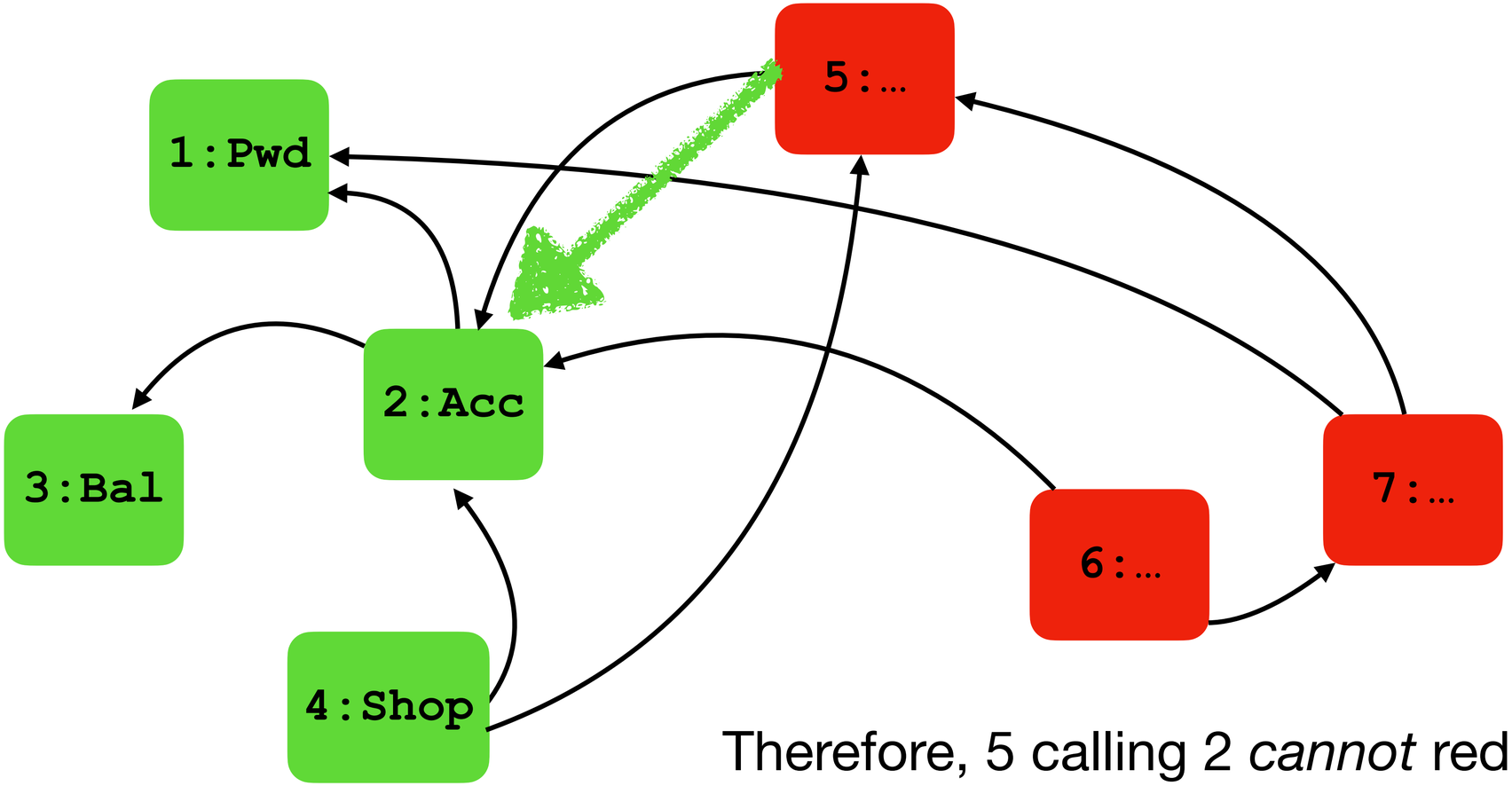

Therefore, 5 calling 2 *cannot* reduce 2.balance.

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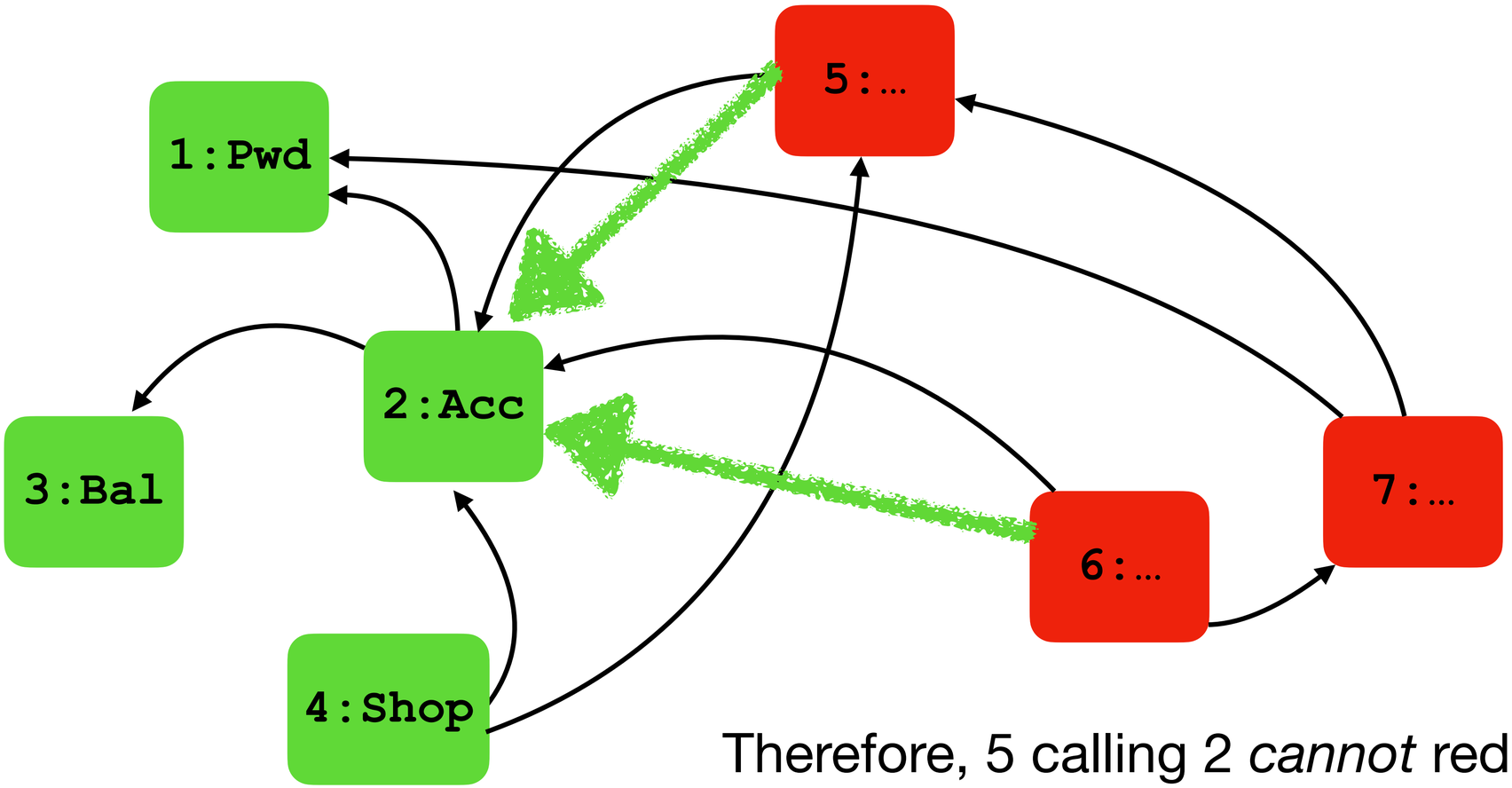
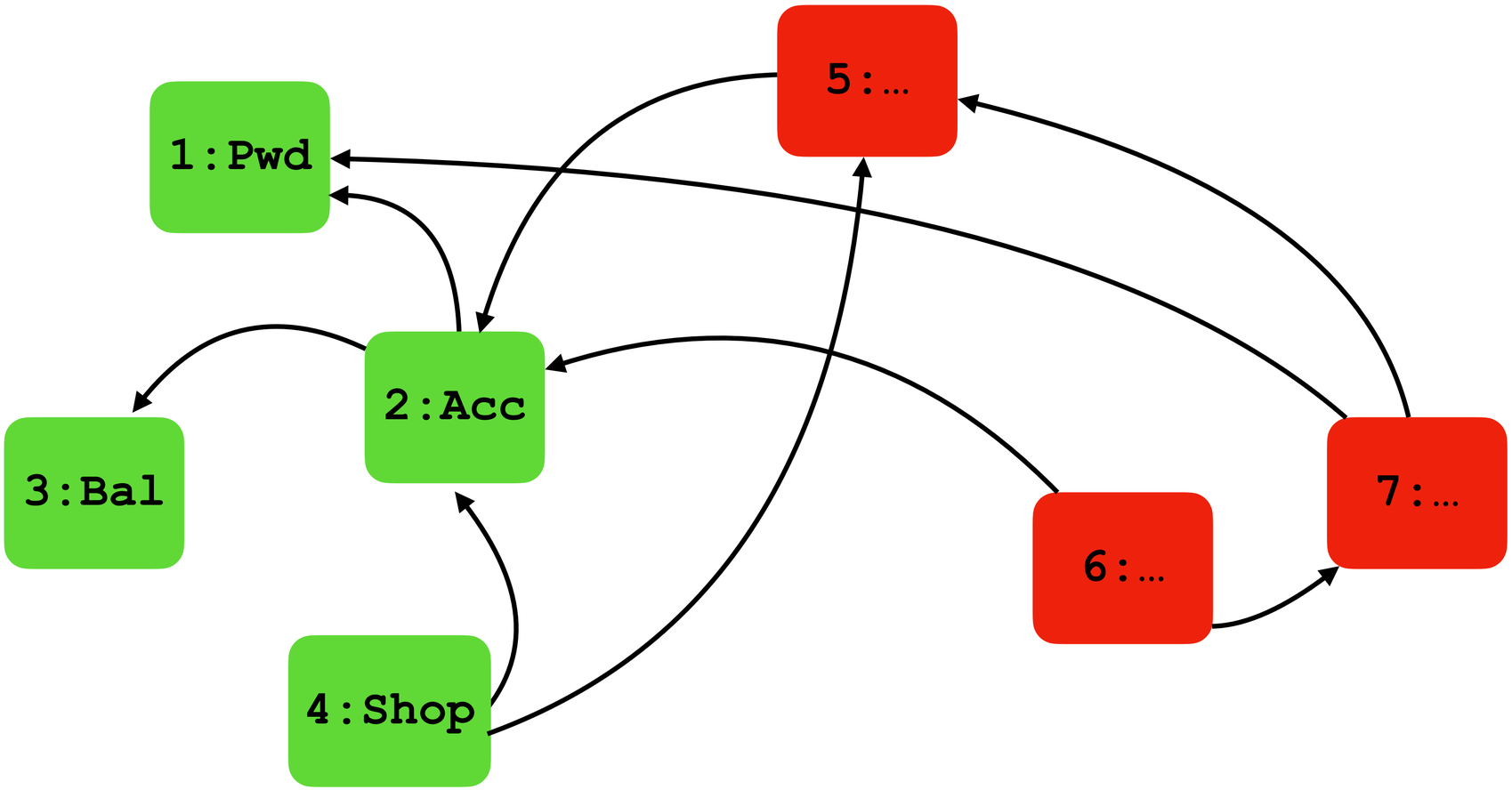


Therefore, 5 calling 2 *cannot* reduce 2.balance.

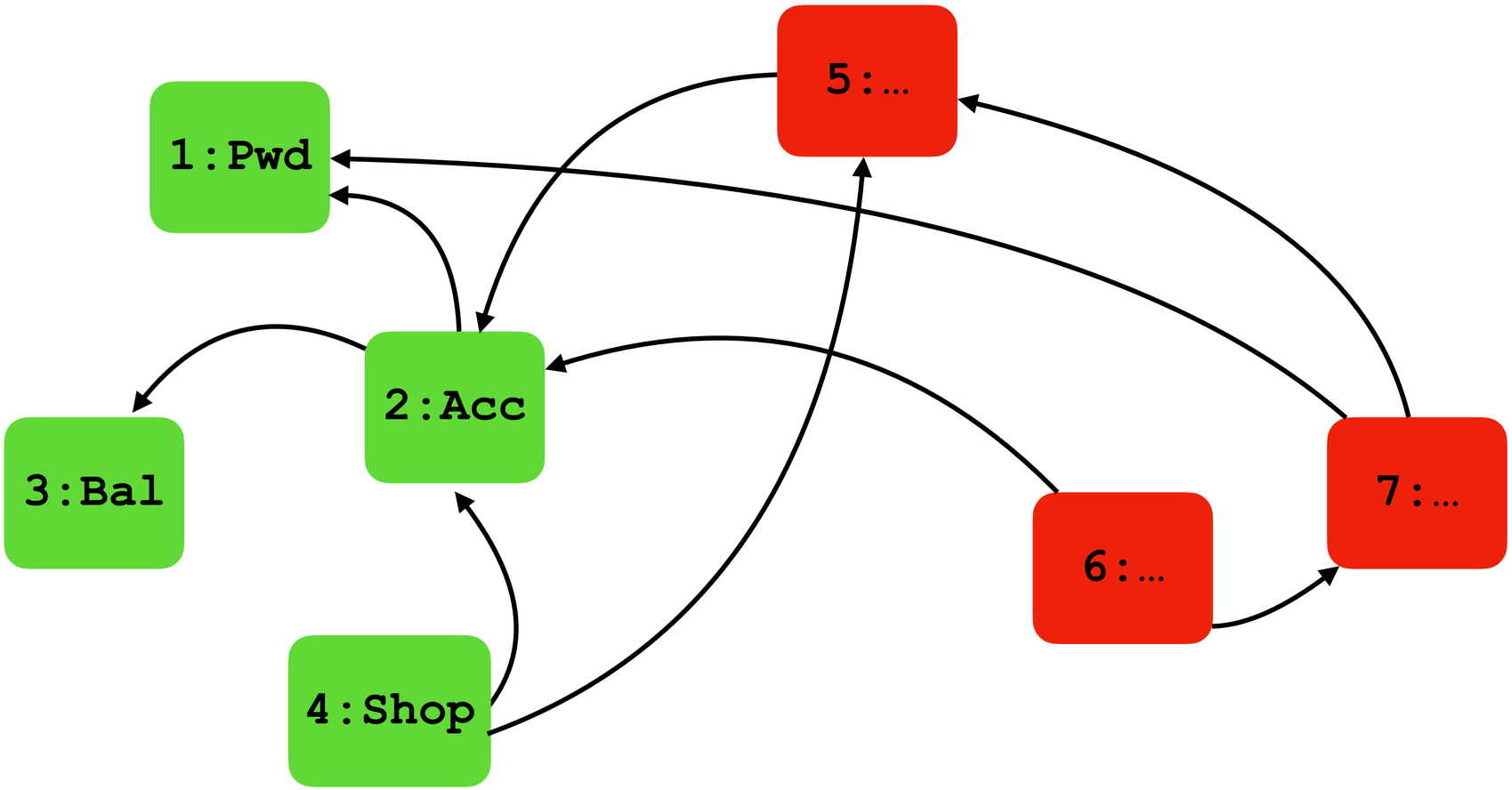
But, 6 calling 2 *may* reduce 2.balance.

Our remit:



Our remit:

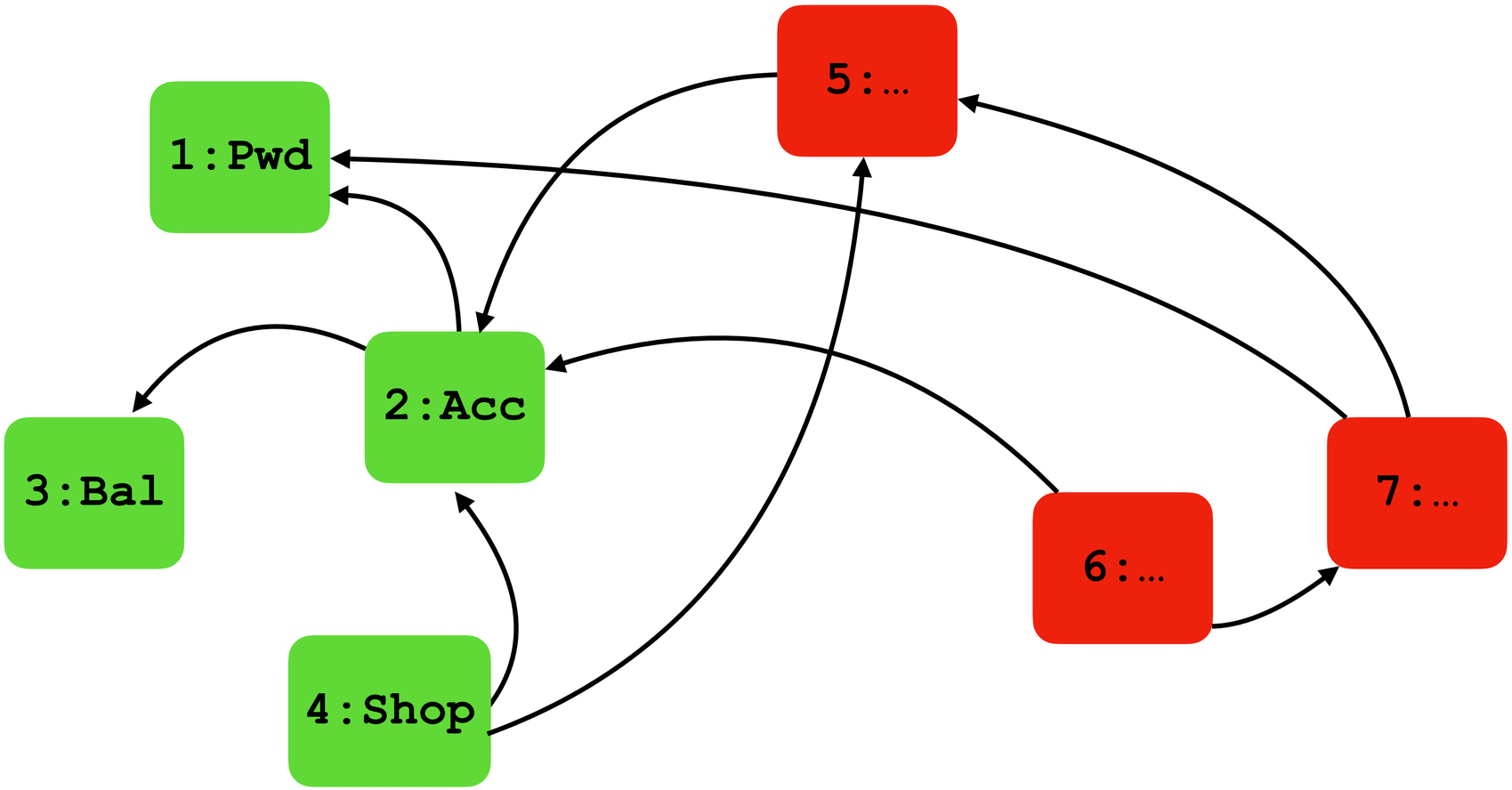
R2: Prove module's adherence to specification — later.



S2: without access to password no decrease in balance

Our remit:

R3: Prove calls from internal to external object.



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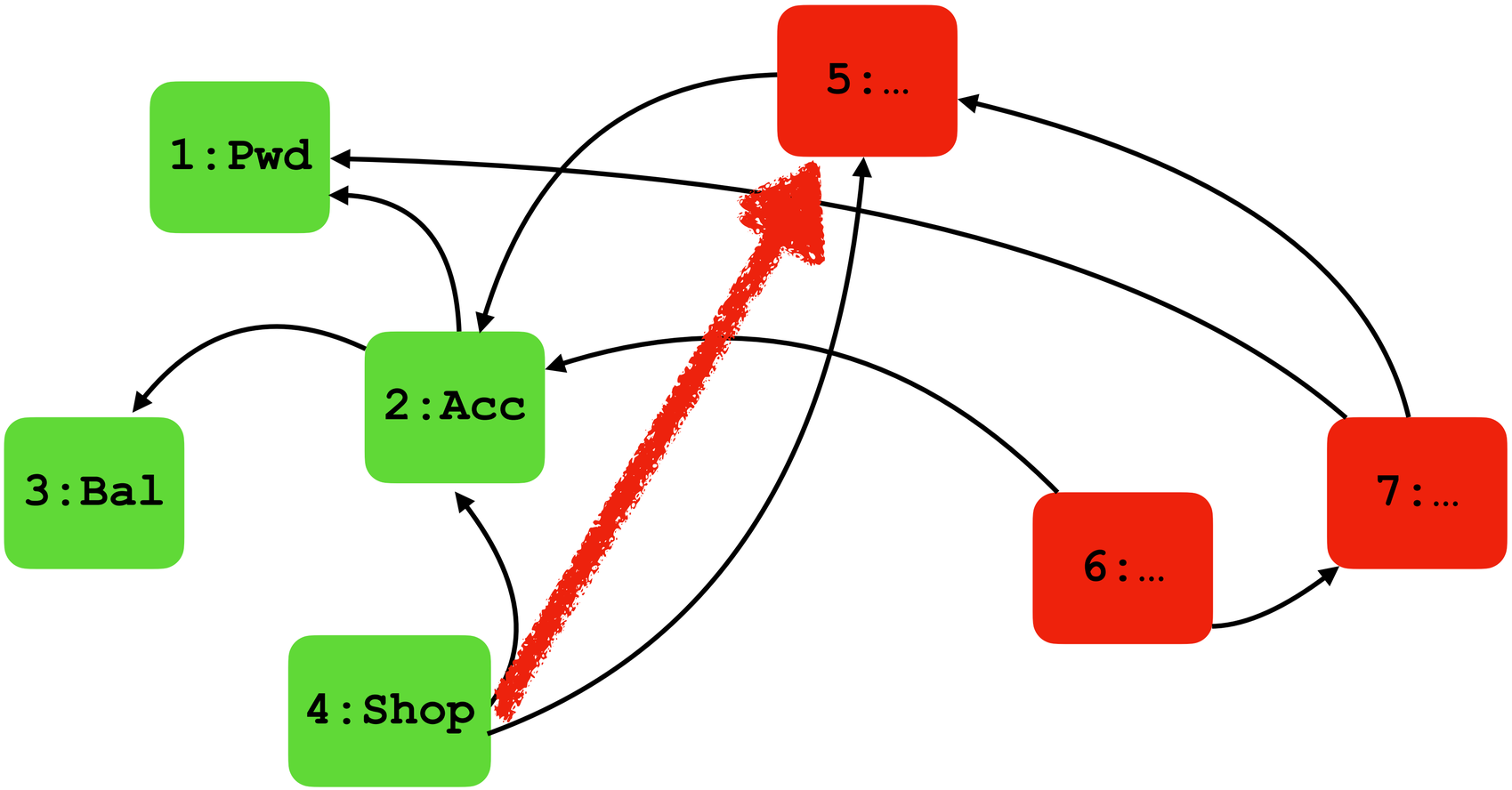

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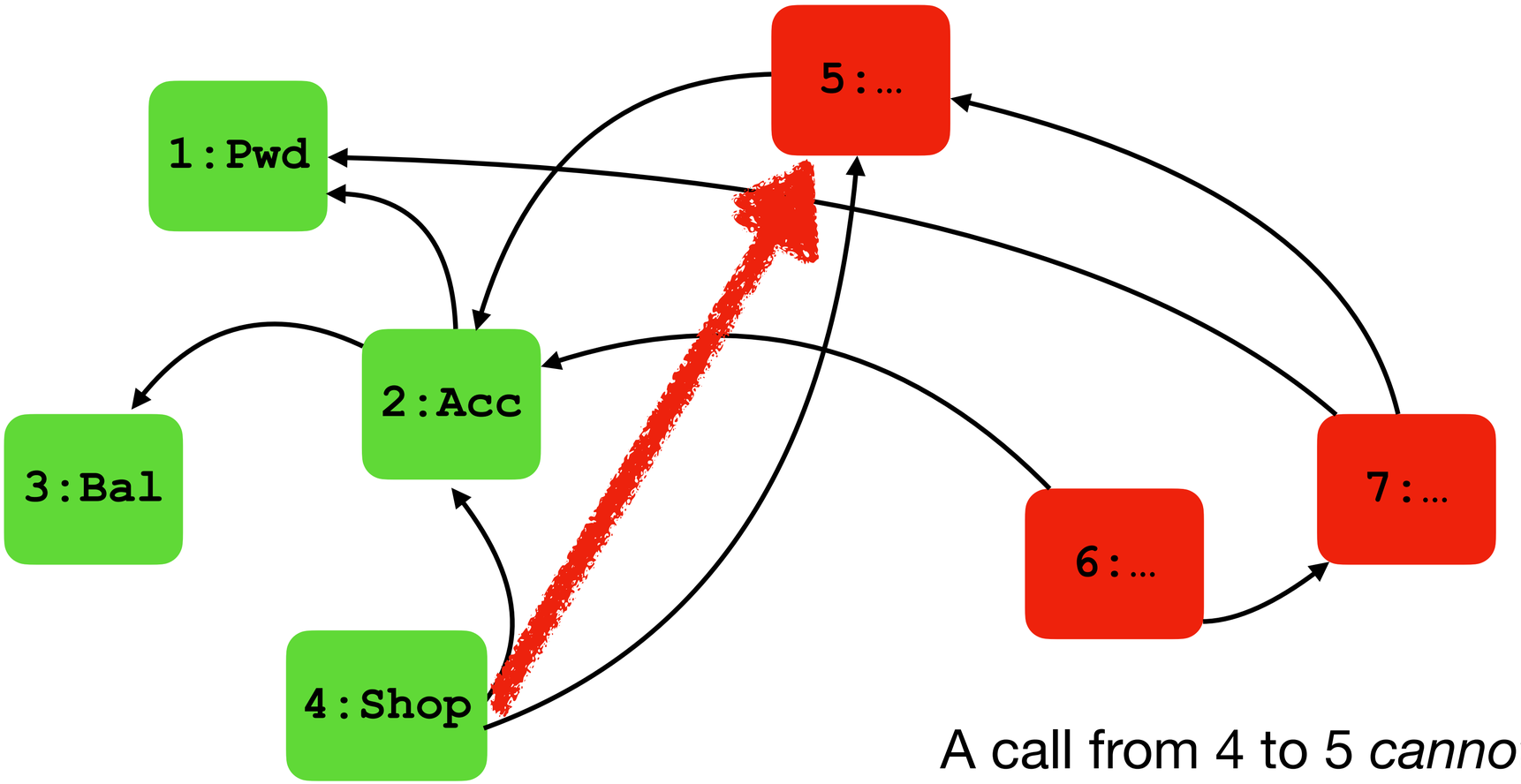

A call from 4 to 5 *cannot* reduce 2.balance

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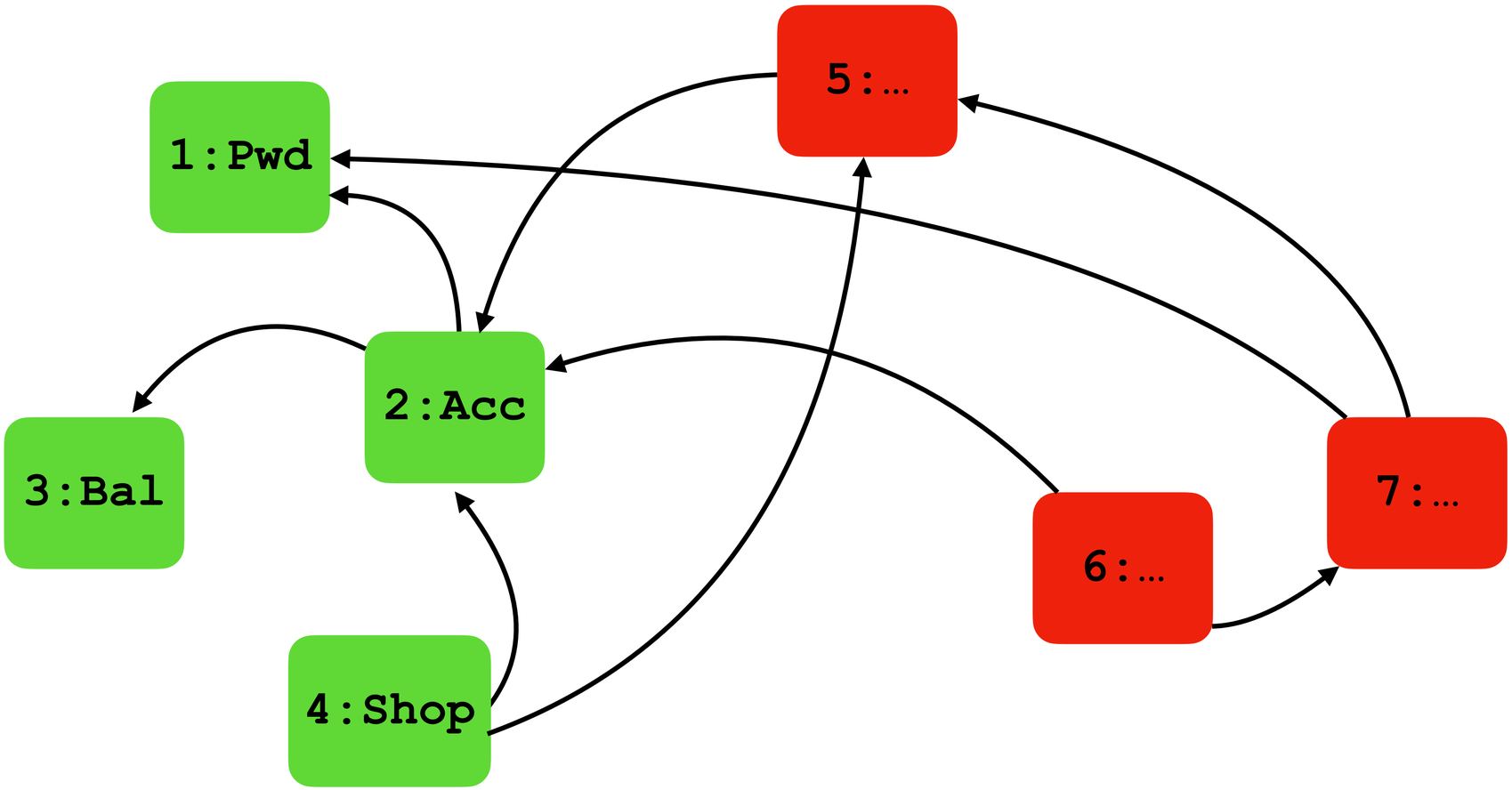

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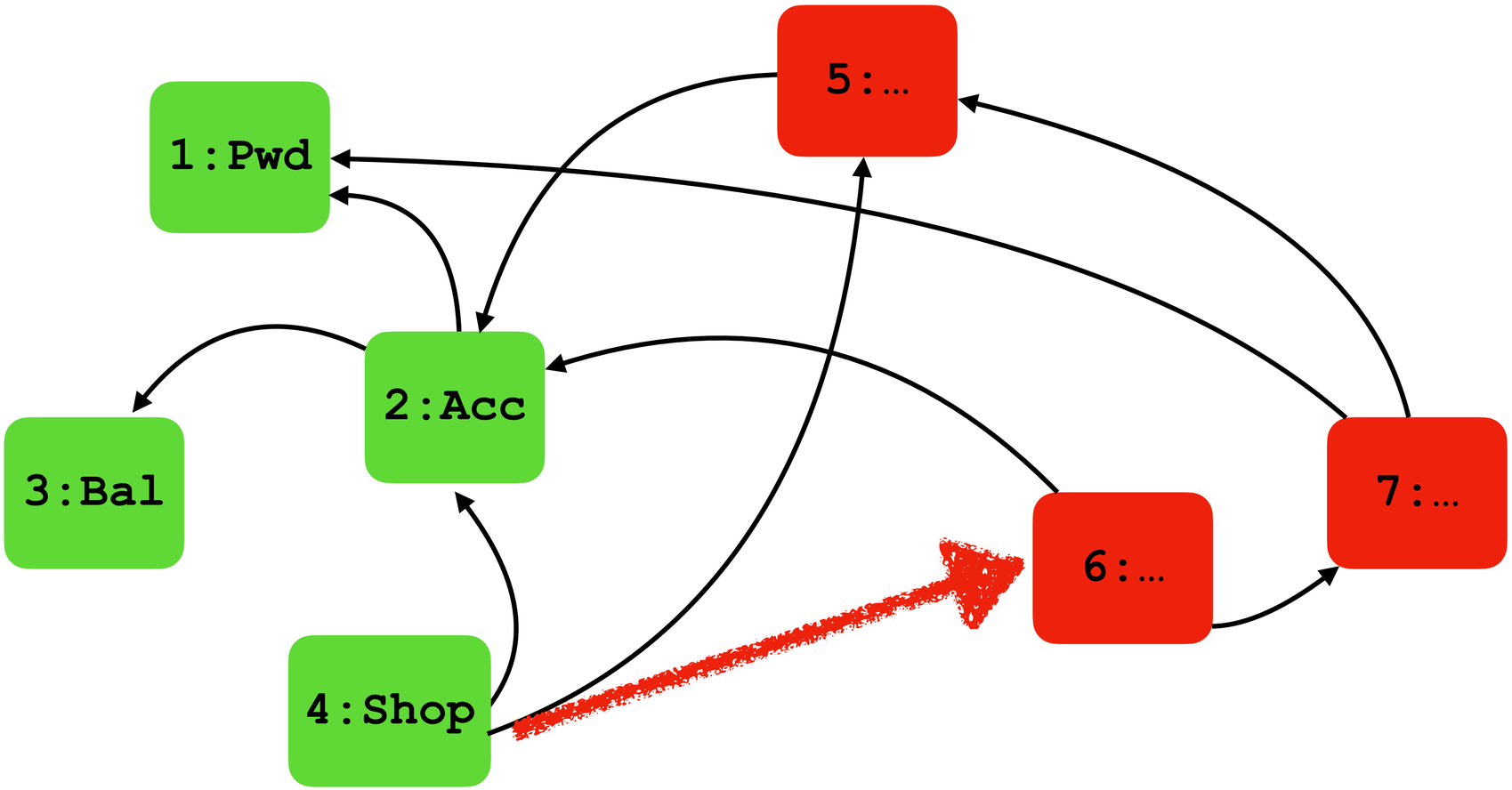

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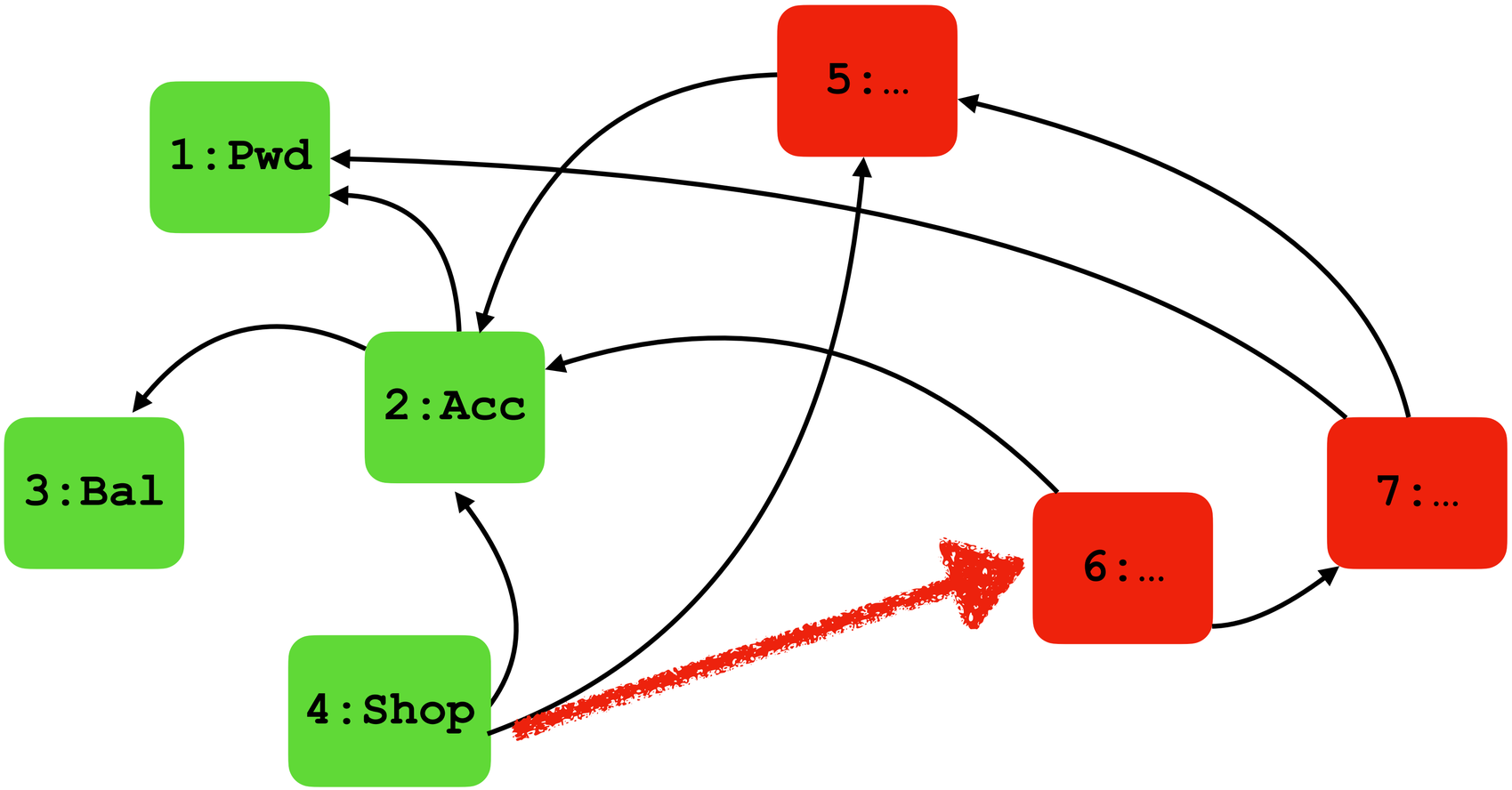
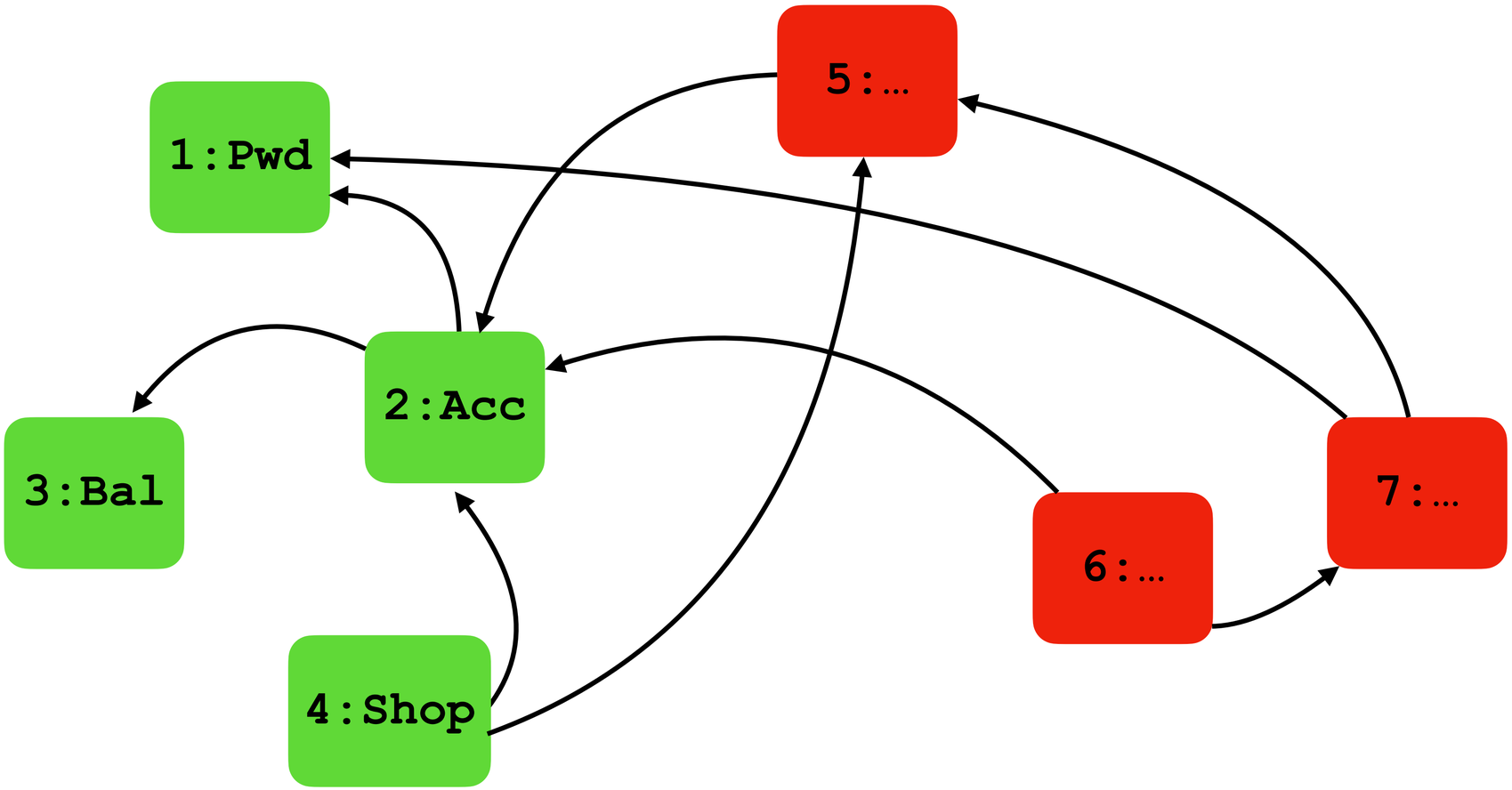
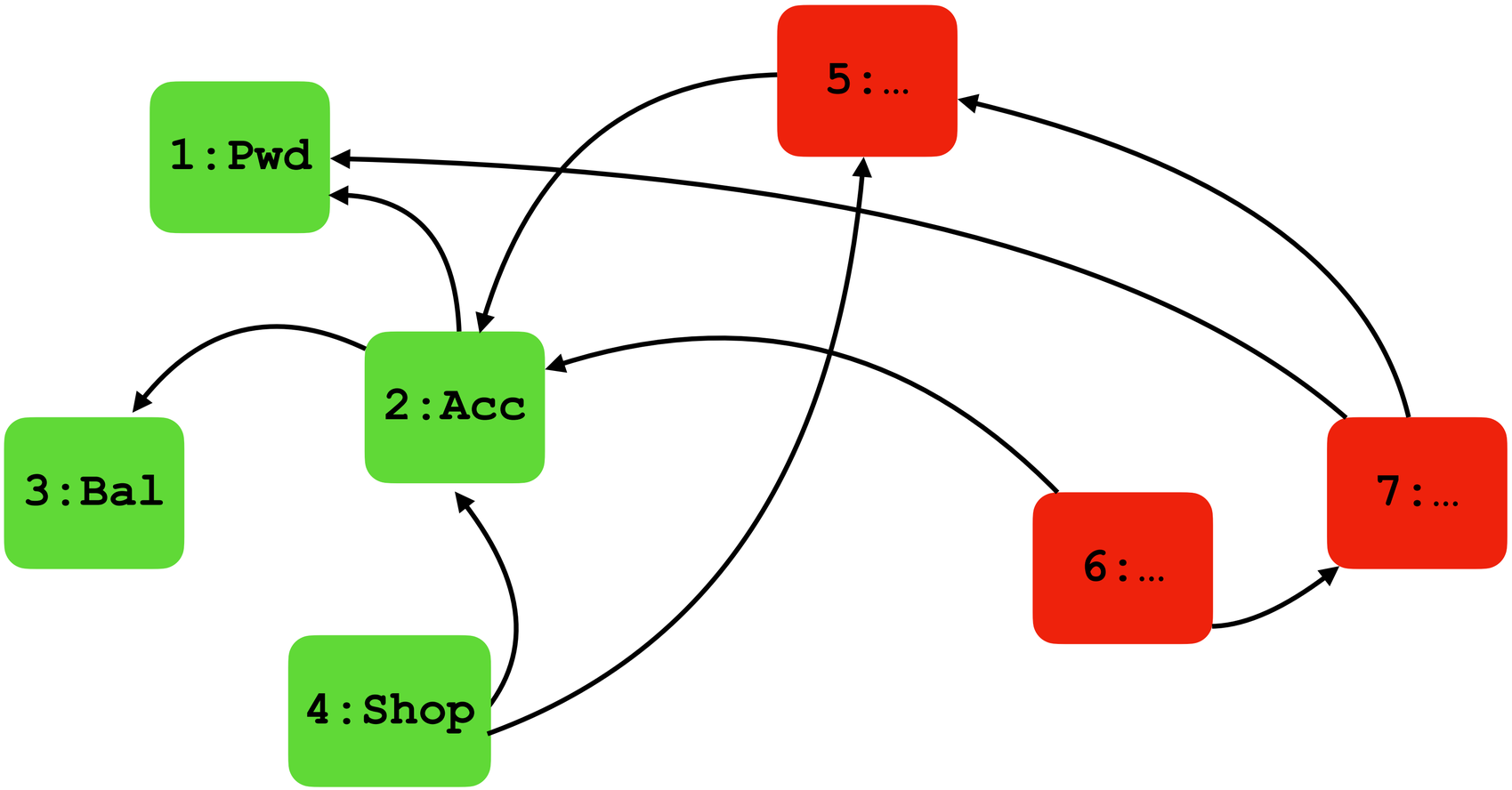


A call from 4 to 6 *might* reduce 2.balance


5 : …
1 : Pwd
2 : Acc
3 : Bal
7 : …
6 : …
4 : Shop

***Not* our remit:** Forbid external access to capabilty.



The Account example in Code

Three Modules

.. assuming all methods are public, and fields are private

```
module M_good
  class Password

  class Account
    field blnce:int
    field pwd: Password
    public method transfer(dest:Account, pwd':Password, amt:int) -> void
      if this.pwd==pwd'
        this.blnce-=amt
        dest.blnce+=amt
     public method set(pwd':Password) -> void
      if this.pwd==null
        this.pwd=pwd'
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module M_bad
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      ... as earlier ...
    public method set(pwd': Password)
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```
module M_fine
  class Password

  class Account
    field blnce:int
    field pwd: Password
    public method transfer(..)
      ... as earlier ...
    public method set(pwd',pwd'': Password)
      if (this.pwd==pwd')
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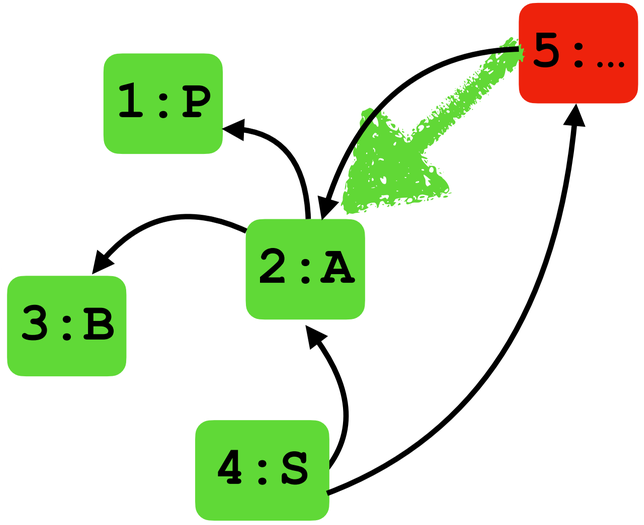


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  class Account
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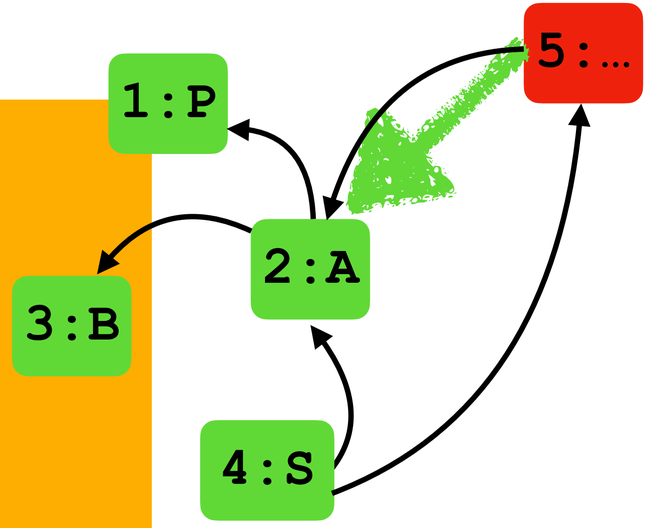
    field pwd: Passwor
    public method trans
      if this.pwd==pwd
        this.blnce-=amt
        dest.blnce+=amt
     public method set
      if this.pwd==nul
        this.pwd=pwd'
```

Remit_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{fine} \models S$

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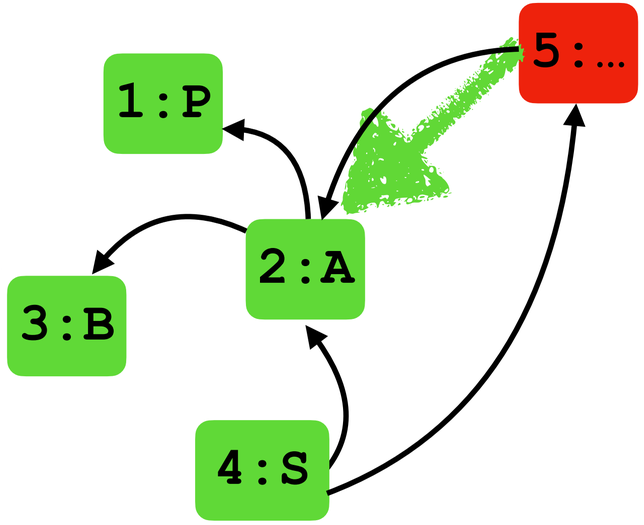


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      if (this.pwd==pwd')
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```

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module Mgood
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  class Account
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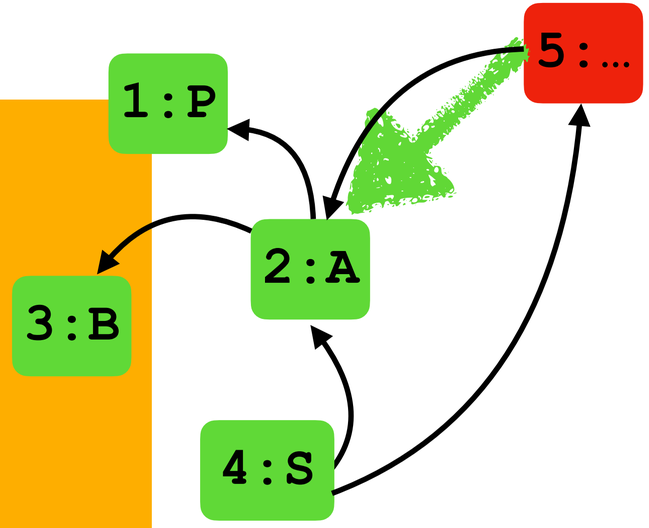
    public method trans
      if this.pwd==pwd
        this.blnce-=amt
        dest.blnce+=amt
     public method set
      if this.pwd==nul
        this.pwd=pwd'
```

Remit_2: A module spec S, such that

$M_{good} \vdash S$

$M_{bad} \nvdash S$

$M_{fine} \vdash S$

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  class Account
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    field pwd: Password
    public method transfer(..)
      ... as earlier ...
    public method set(pwd',pwd'': Password)
      if (this.pwd==pwd')
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```

```
1  class Shop
2    field accnt :Account, invntry: Inventory, clients: [external]
3
4    public method buy(buyer: external, anItem: Item) -> void
5      int price = anItem.price
6      int oldBlnce = this.accnt.blnce
7      buyer.payMe(this.accnt, price)
8      if (this.accnt.blnce == oldBlnce+price)
9         this.send(buyer,anItem)
10     else
11        buyer.tell("you have not paid me")
12     private method send(buyer: external anItem: Item) -> void
13        ...
```

```
1  class Shop
2    field accnt :Account, invntry: Inventory, clients: [
3
4    public method buy(buyer: external, anItem: Item)
5      int price = anItem.price
6      int oldBlnce = this.accnt.blnce
7      buyer.payMe(this.accnt, price)
8      if (this.accnt.blnce == oldBlnce+price)
9         this.send(buyer,anItem)
10     else
11        buyer.tell("you have not paid me")
12     private method send(buyer: external anItem: Item) -> void
13        ...
```

```
1  class Shop
2    field accnt :Account, invntry: Inventory, clients: [
3
4    public method buy(buyer: external, anItem: Item)
5      int price = anItem.price
6      int oldBlnce = this.accnt.blnce
7      buyer.payMe(this.accnt, price)
8      if (this.accnt.blnce == oldBlnce+price)
9         this.send(buyer,anItem)
10     else
11        buyer.tell("you have not paid me")
12     private method send(buyer: external anItem: Item) -> void
13        ...
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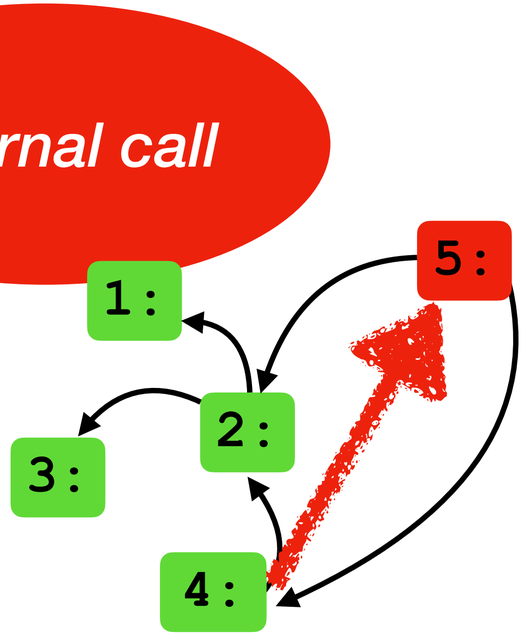

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```
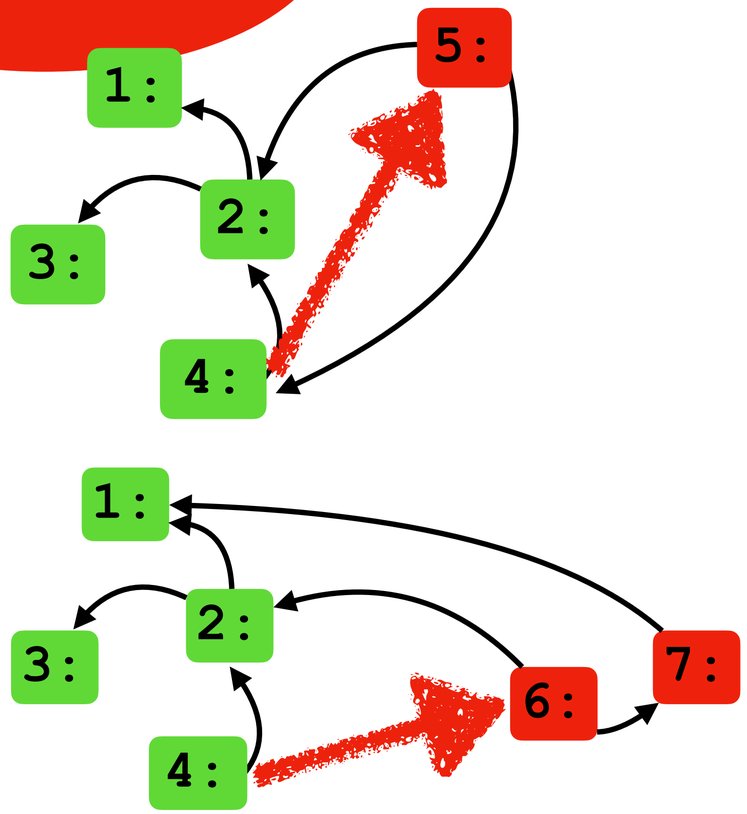

External call



```
1  class Shop
2    field accnt :Account, invntry: Inventory, clients: [
3
4    public method buy(buyer: external, anItem: Item)
5      int price = anItem.price
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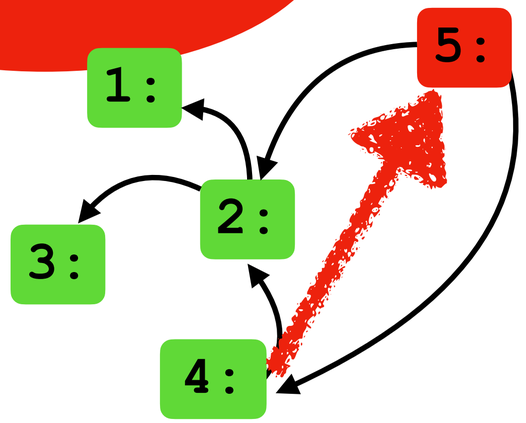
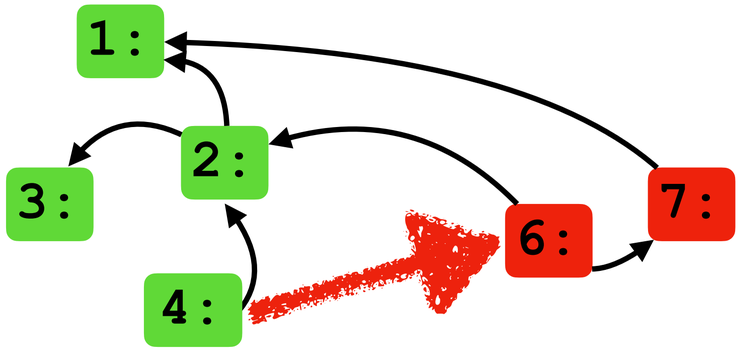
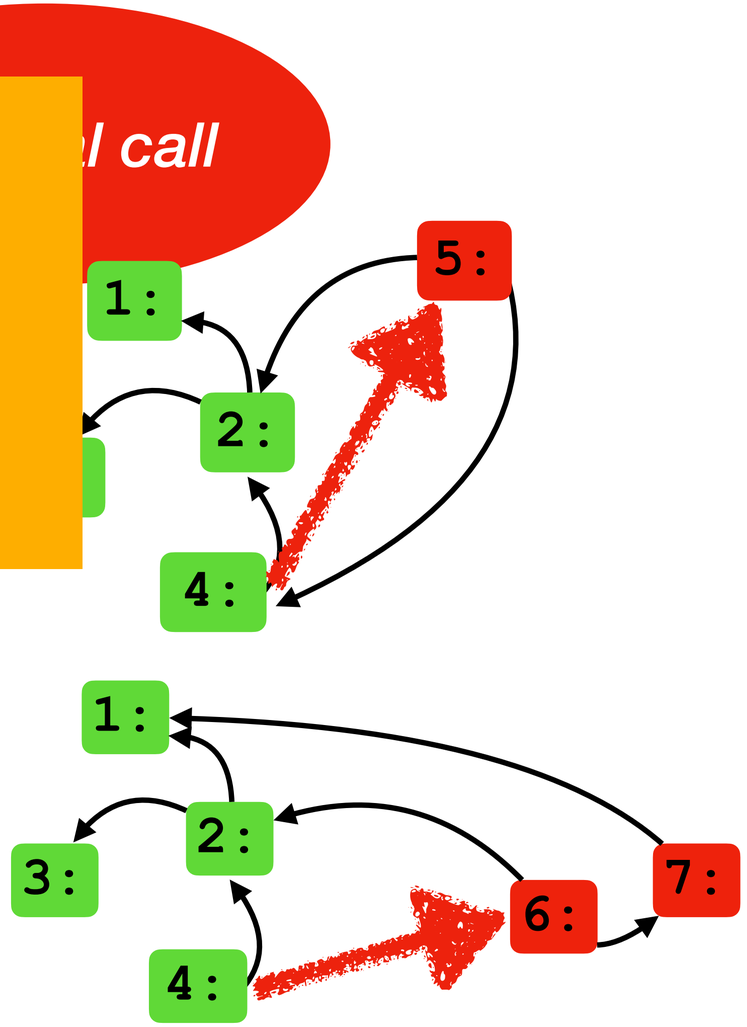




If `Account` comes from a “good” module,
and `buyer` has no unprotected access to `4.accnt.pwd`,
then
`4.accnt.blnce` will not decrease `4.accnt.blnce`,

```
1  class Shop
2    field accnt :A
3
4    public method
5      int price =
6      int oldBlnce
7      buyer.payMe(
8      if (this.acc
9         this.send
10     else
11        buyer.tell("you have not paid me")
12     private method send(buyer: external anItem: Item) -> void
13        ...
```

Remit_3: An inference system, such that we can prove **external** calls.



If `Account` comes from a “good” module,
and `buyer` has no unprotected access to `4.accnt.pwd`,
then
`4.accnt.blnce` will not decrease `4.accnt.blnce`,

Remit_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{better} \models S$

Remit_1: A module spec S, such that

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Remember: A capability represents a transferable right
to perform one or more operations on a given object

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$M_{good} \models S$
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Remember: A capability represents a transferable right
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So: “The password enables withdrawal from the account”?

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~~***So***: "The password enables withdrawal from the account"?~~

Or: "Without the password call of withdraw will fail"?

Remit_1: A module spec S, such that

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Remember: A capability represents a transferable right
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~~***So***: "The password enables withdrawal from the account"?~~

~~***Or***: "Without the password call of withdraw will fail"?~~

Or: "Without eventual access to password no reduction of the balance of the account"?

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So: $\forall$ a:Accnt,b:Num. { "without eventual external access to" a.pwd $\wedge$ a.balance$\geq$b}

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$M_{bad} \not\models S$
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Remember: A capability represents a transferable right
to perform one or more operations on a given object

R1: Specify that external access to capability necessary for effect.

~~***So***: "The password enables withdrawal from the account"?~~

~~***Or***: "Without the password call of withdraw will fail"?~~

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In general: $\forall$ x1:C1,x2:C2…{ A }

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Remit_1_a : Meaning of “without eventual external access to”

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Remit_1_a : Meaning of “without eventual external access to”

Remit_1_b : Meaning of ∀ x1:C1,x2:C2… { A }

Remit_1: A module spec S, such that …

o: ∀ a:Accnt. ∀ b:Num. { “without eventual external access to” a.pwd ∧ a.balance≧b}

In general: ∀ x1:C1,x2:C2…{ A }

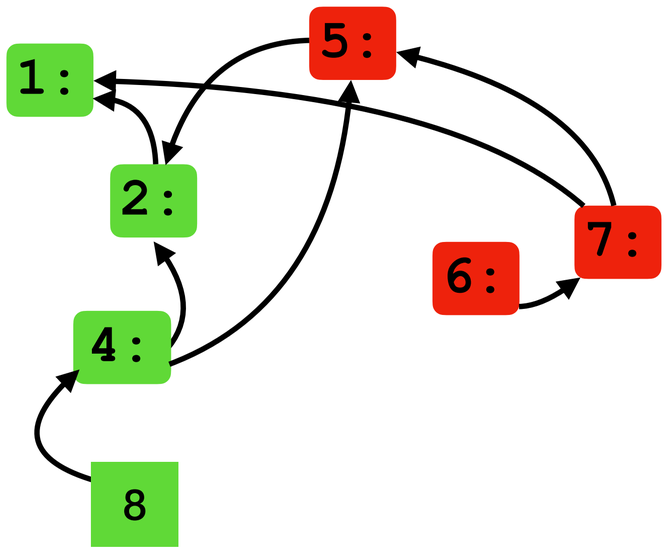
Remit_1_a : Meaning of “without eventual external access to”

Remit_1_b : Meaning of ∀ x1:C1,x2:C2… { A }

Remit_1_a : Meaning of “without eventual external access to”

… is about an external object eventually obtainining access.

Def: 《o》 ≜ ∀ o’ [o’ external and reachable from top of stack frame ⇒ 《o》↛o’]

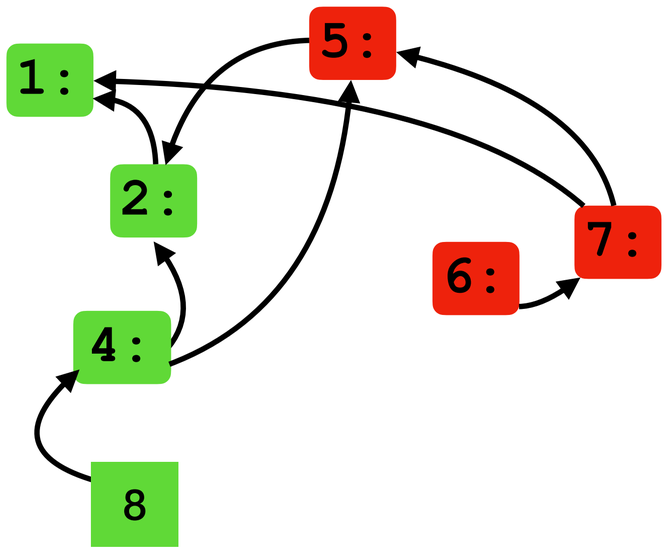


Remit_1_a : Meaning of “without eventual external access to”

… is about an external object eventually obtainining access.

Def: 《o》⊣o’ ≜ the penultimate object on any path from o’ to o is internal.

Def: 《o》 ≜ ∀ o’ [o’ external and reachable from top of stack frame ⇒ 《o》⊣o’]



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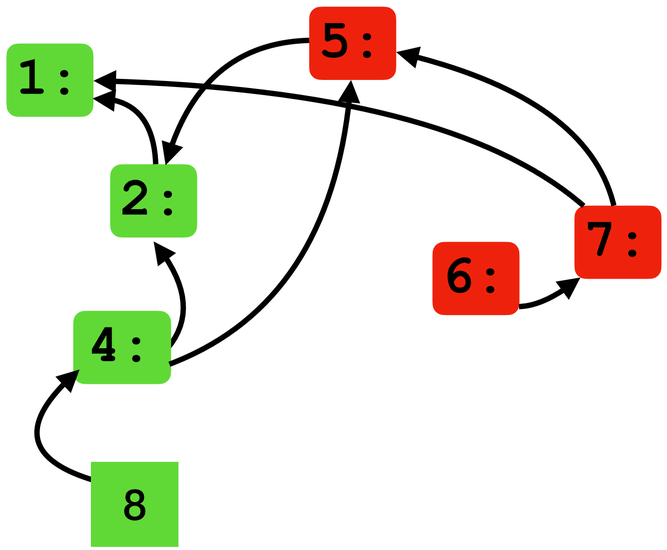
For example:



Remit_1_a : Meaning of “without eventual external access to”

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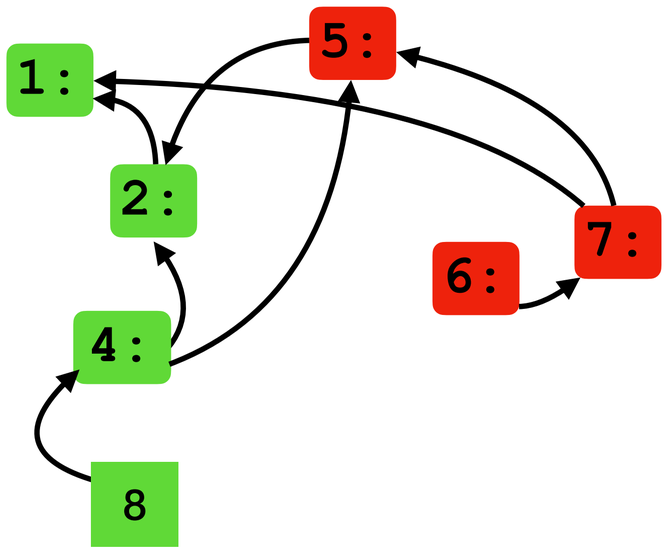
Def: $\langle\!\langle o \rangle\!\rangle \dashv o'$ $\triangleq$ the penultimate object on any path from o’ to o is internal.

Def: $\langle\!\langle o \rangle\!\rangle$ $\triangleq$ $\forall$ o’ [o’ external and reachable from top of stack frame $\Rightarrow$ $\langle\!\langle o \rangle\!\rangle \dashv o'$]

For example:



$\ldots \models \langle\!\langle 1 \rangle\!\rangle \dashv 5$

$\ldots \not\models \langle\!\langle 1 \rangle\!\rangle \dashv 6$

$\ldots \not\models \langle\!\langle 2 \rangle\!\rangle \dashv 4$

$\ldots \not\models \langle\!\langle 2 \rangle\!\rangle \dashv 8$

Remit_1_a : Meaning of “without eventual external access to”

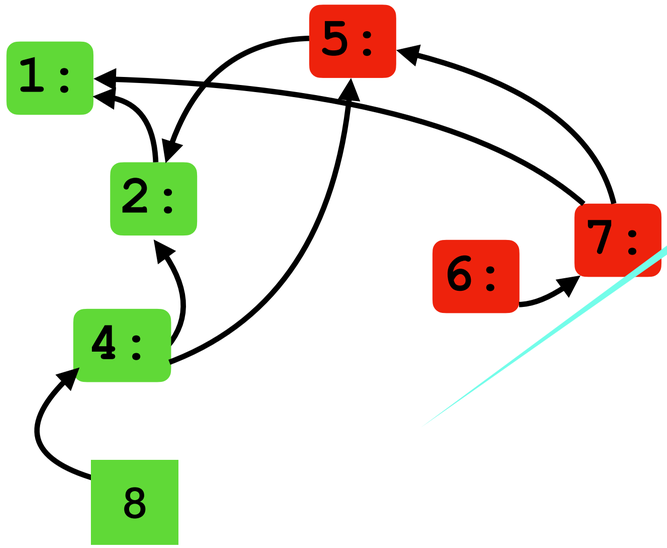
… is about an external object eventually obtainining access.

Def: 《o》+o’ ≜ the penultimate object on any path from o’ to o is internal.

Def: 《o》 ≜ ∀ o’ [o’ external frame ⇒ 《o》+o’]

Protection is “relative”
to an object

For example:



… ⊨ 《1》+ 5

… ⊭ 《1》+ 6

… ⊭ 《2》+ 4

… ⊭ 《2》+ 8

Remit_1_a : Meaning of "without eventual external access to"

Def: 《o》⫞o' ≜ the penultimate object on any path from o' to o is internal.

```
1  class Shop
2    field accnt :Account, invntry: Inventory, clients: [external]
3
4    public method buy(buyer: external, anItem: Item) -> void
5      int price = anItem.price
6      int oldBlnce = this.accnt.blnce
7      buyer.payMe(this.accnt, price)
8      if (this.accnt.blnce == oldBlnce+price)
9         this.send(buyer,anItem)
10     else
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13        ...
```

Remit_1_a : Meaning of "without eventual external access to"

Def: 《o》⊰o' ≜ the penultimate object on any path from o' to o is internal.

PRE: 《this.accnt.pwd》⊰buyer ∧ this.accnt.blnce == b

POST: this.accnt.blnce >= b

```
1  class Shop
2    field accnt :Account, invntry: Inventory, clients: [external]
3
4    public method buy(buyer: external, anItem: Item) -> void
5      int price = anItem.price
6      int oldBlnce = this.accnt.blnce
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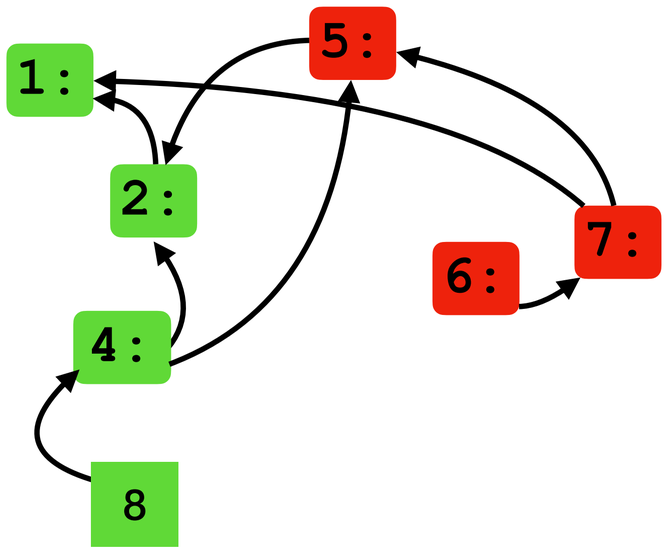

… ⊨ 《1》⊣ 5

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… ⊭ 《2》⊣ 4

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Def: $\langle\!\langle o \rangle\!\rangle \dashv o'$ $\triangleq$ the penultimate object on any path from o’ to o is internal.

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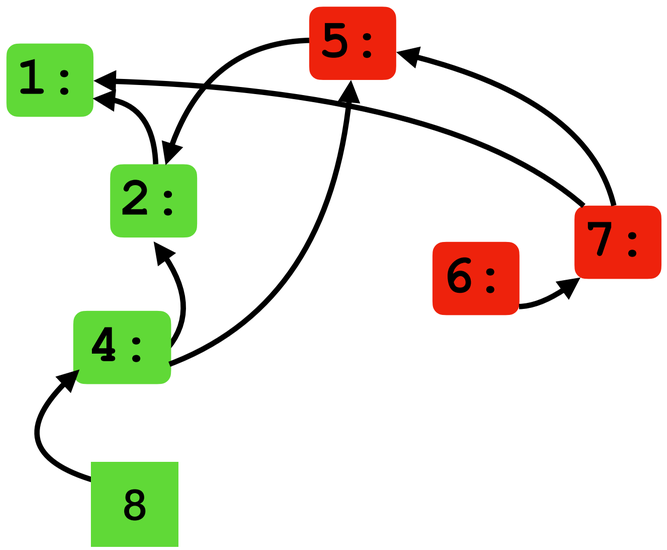

$\ldots \models \langle\!\langle 1 \rangle\!\rangle \dashv 5$

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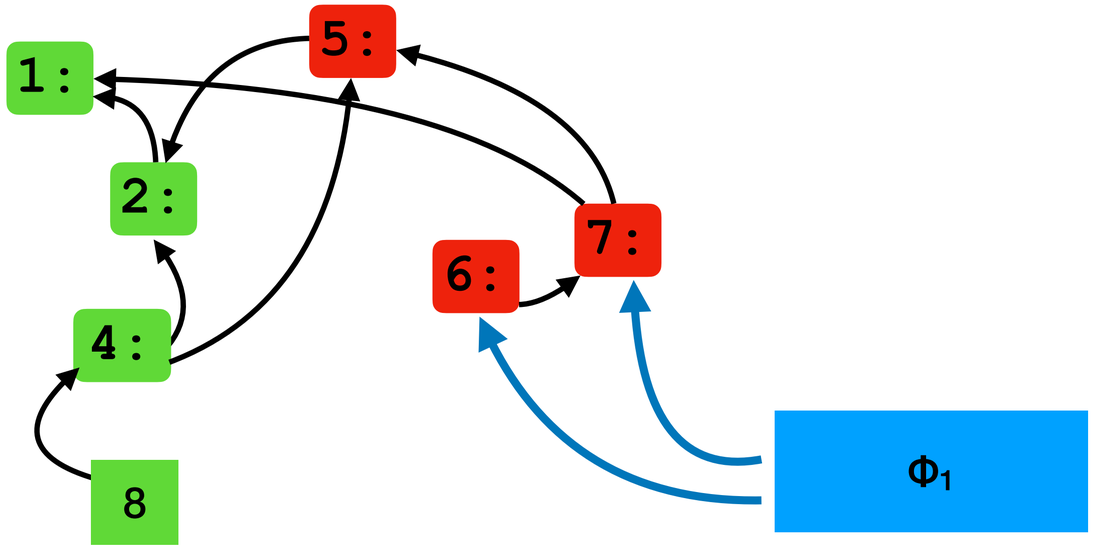

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$\Phi_1 \ldots \not\models \langle\!\langle 1 \rangle\!\rangle$

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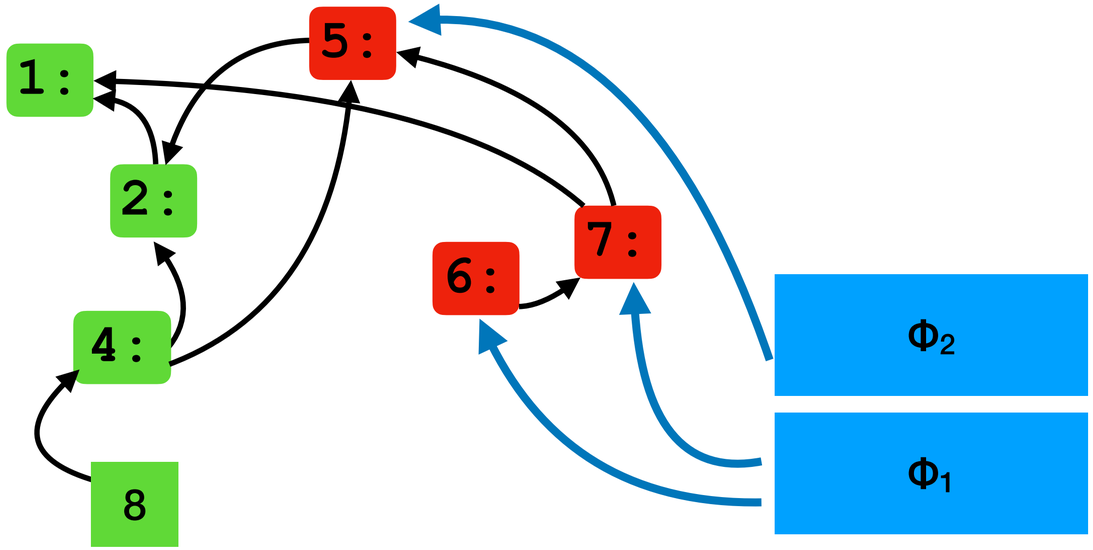

$\ldots \models \langle\!\langle 1 \rangle\!\rangle \dashv 5$

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$\Phi_1 \Phi_2 \ldots \models \langle\!\langle 1 \rangle\!\rangle$

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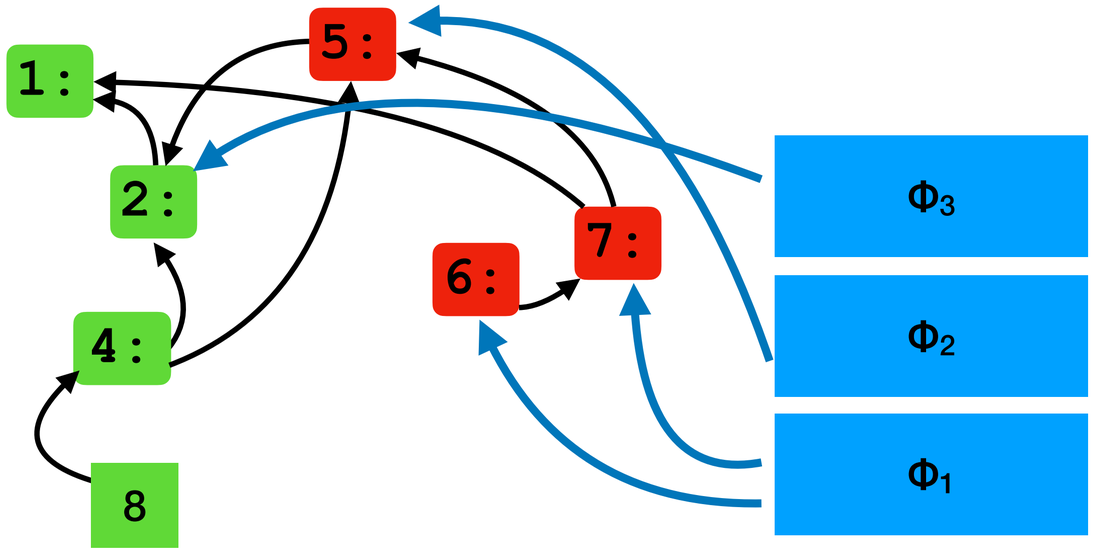

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$\Phi_1 \ldots \not\models \langle\!\langle 1 \rangle\!\rangle$

$\Phi_1\Phi_2 \ldots \models \langle\!\langle 1 \rangle\!\rangle$

$\Phi_1\Phi_2 \ldots \not\models \langle\!\langle 2 \rangle\!\rangle$

$\Phi_1\Phi_2\Phi_3 \ldots \models \langle\!\langle 1 \rangle\!\rangle$

$\Phi_1\Phi_2\Phi_3 \ldots \models \langle\!\langle 2 \rangle\!\rangle$

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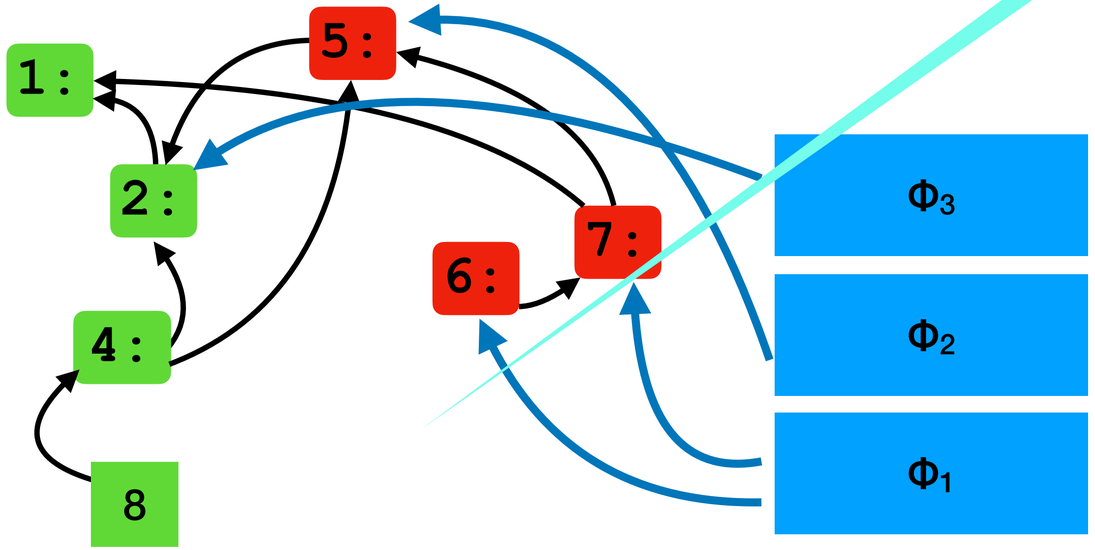
For example:



… ⊨ 《1》⊣ 5

… ⊭ 《1》⊣ 6

… ⊭ 《2》⊣ 4

… ⊭ 《2》⊣ 8

Φ1… ⊭ 《1》

Φ1Φ2… ⊨ 《1》

Φ1Φ2… ⊭ 《2》

Φ1Φ2Φ3… ⊨ 《1》

Φ1Φ2Φ3… ⊨ 《2》

Remit_1_b : Meaning of ∀x1:C1,x2:C2… { A }

Definition

$$M \models \forall x1:C1,\ldots xn:Cn\ \{ A \}$$

$$\triangleq$$

Remit_1_b : Meaning of ∀x1:C1,x2:C2… { A }

Definition

$$M \models \forall x1{:}C1,\ldots xn{:}Cn\ \{ A \}$$

$$\triangleq$$

$$\forall M', \forall \sigma, \sigma', \forall \alpha 1,\ldots \alpha n[$$

$$]$$

Remit_1_b : Meaning of $\forall$x1:C1,x2:C2… { A }

Definition

$$M \models \forall x1:C1,\ldots xn:Cn \{ A \}$$

$$\triangleq$$

$$\forall M', \forall \sigma, \sigma', \forall \alpha 1,\ldots \alpha n[$$

$$M, \sigma \models \texttt{this}: \text{ext} \wedge \alpha 1 :C1,\ldots \alpha n:Cn \wedge A[\alpha 1,\ldots \alpha n / x1,..xn]$$

$$]$$

Remit_1_b : Meaning of $\forall$x1:C1,x2:C2… { A }

Definition

$$M \models \forall x1{:}C1,\ldots xn{:}Cn\ \{ A \}$$

$$\triangleq$$

$$\forall M', \forall \sigma, \sigma', \forall \alpha 1,\ldots \alpha n[$$

$$M, \sigma \models \texttt{this}: \text{ext} \ \wedge\ \alpha 1 : C1,\ldots \alpha n{:}Cn \ \wedge A[\ \alpha 1,\ldots \alpha n \ / \ x1,..xn]$$

$$\wedge$$

$$M'^{*}M, \sigma \rightsquigarrow^{*} \sigma'$$

$$]$$

Remit_1_b : Meaning of $\forall$x1:C1,x2:C2… { A }

Definition

$$M \models \forall x1{:}C1,\ldots xn{:}Cn\ \{ A \}$$

$$\triangleq$$

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$$M, \sigma \models \texttt{this}: \text{ext} \ \wedge\ \alpha 1 : C1,\ldots \alpha n{:}Cn \ \wedge A[\ \alpha 1,\ldots \alpha n\ /\ x1,..xn]$$

$$\wedge$$

$$M'^{*}M, \sigma \rightsquigarrow^{*} \sigma'$$

$$\Rightarrow$$

$$M, \sigma' \models \texttt{this}: \text{ext} \longrightarrow A[\ \alpha 1,\ldots \alpha n\ /\ x1,..xn]$$

$$]$$

Remit_1_b : Meaning of ∀x1:C1,x2:C2… { A }

Definition

Scoped execution

$$M \models \forall x1{:}C1,\ldots xn{:}Cn\ \{ A \}$$

$$\triangleq$$

$$\forall M', \forall \sigma, \sigma', \forall \alpha 1,\ldots \alpha n[$$

$$M, \sigma \models \texttt{this}: \text{ext} \wedge \alpha 1 :C1,\ldots \alpha n{:}Cn \wedge A[\alpha 1,\ldots \alpha n / x1,..xn]$$

$$\wedge$$

$$M'{*}M, \sigma \rightsquigarrow^{*} \sigma'$$

$$\Rightarrow$$

$$M, \sigma' \models \texttt{this}: \text{ext} \longrightarrow A[\alpha 1,\ldots \alpha n / x1,..xn]$$

$$]$$

Remit_1_c : Meaning of $M'^{*}M,\ \sigma \rightsquigarrow^{*} \sigma'$

Definition

$$M'^{*}M,\ \sigma \rightsquigarrow^{*} \sigma' \triangleq$$

$\longrightarrow$

Remit_1_c : Meaning of $M'*M,\ \sigma \rightsquigarrow^* \sigma'$

Definition

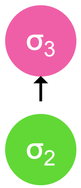
$M'*M,\ \sigma \rightsquigarrow^* \sigma' \quad \triangleq \quad M'*M,\ \sigma \to^* \sigma'$ "while not popping the top frame of σ"

$\longrightarrow$

Remit_1_c : Meaning of $M'*M,\ \sigma \rightsquigarrow^* \sigma'$

Definition

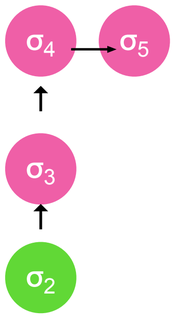
$M'*M,\ \sigma \rightsquigarrow^* \sigma' \quad \triangleq \quad M'*M,\ \sigma \text{ -}>^* \sigma'$ "while not popping the top frame of σ"



Remit_1_c : Meaning of $M'*M,\ \sigma \rightsquigarrow^* \sigma'$

Definition

$M'*M,\ \sigma \rightsquigarrow^* \sigma' \quad \triangleq \quad M'*M,\ \sigma \text{ -}>^* \sigma'$ "while not popping the top frame of σ"



Remit_1_c : Meaning of $M'^*M,\ \sigma \rightsquigarrow^* \sigma'$

Definition

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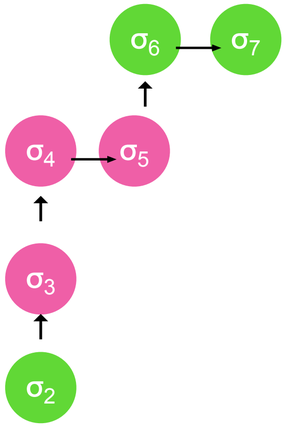

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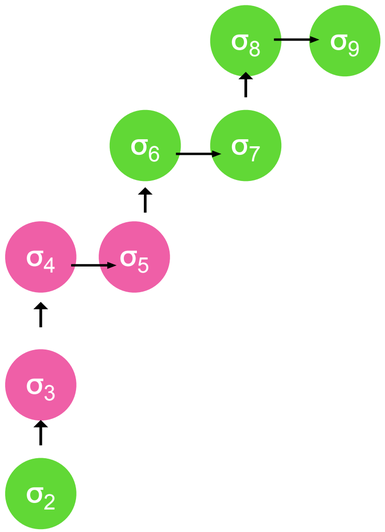

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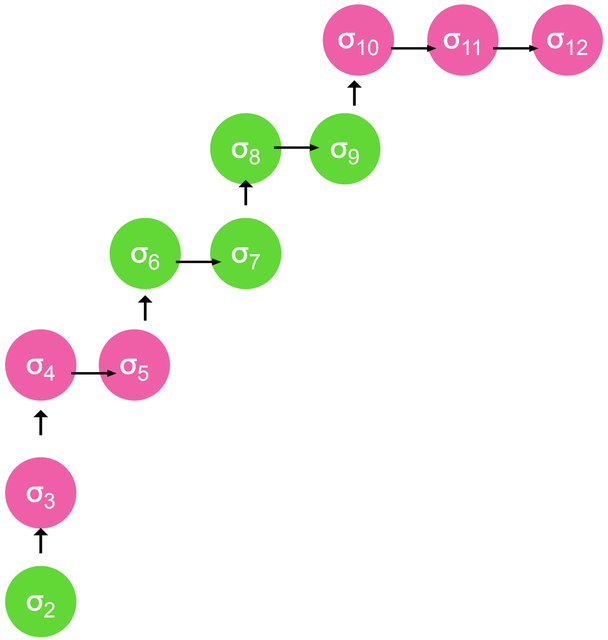

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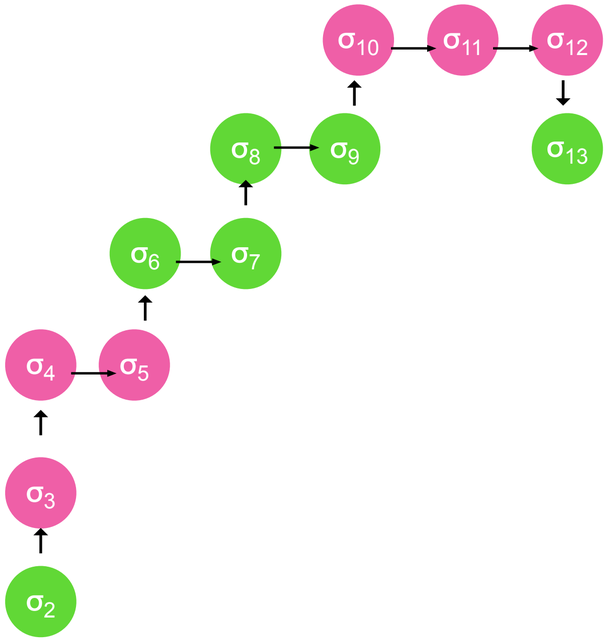

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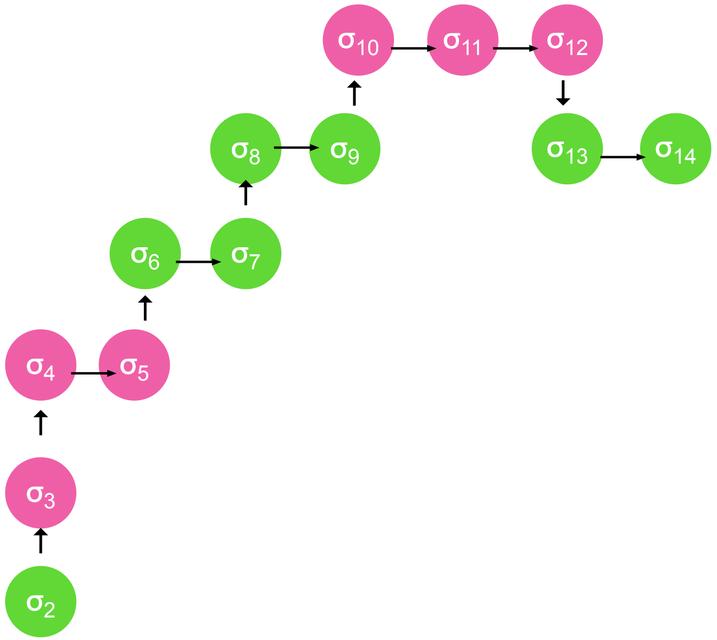

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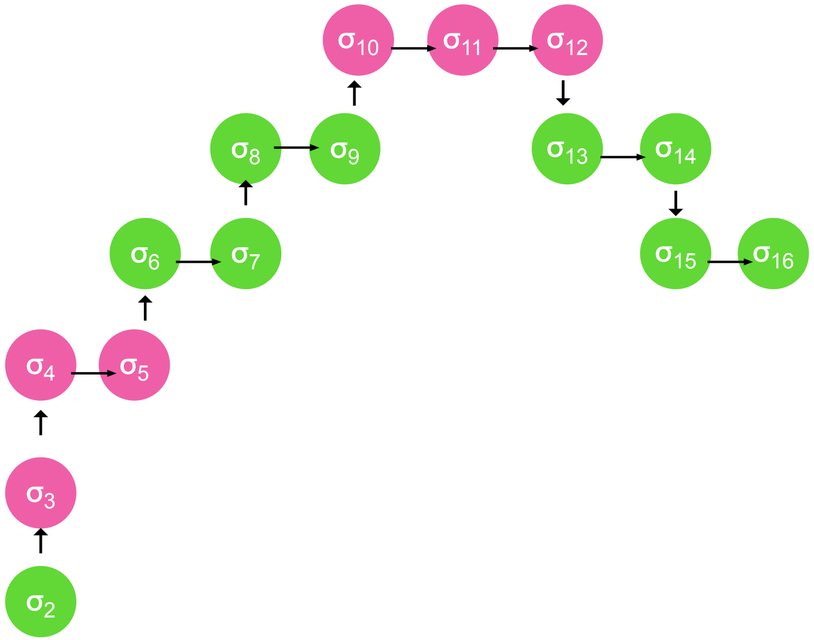

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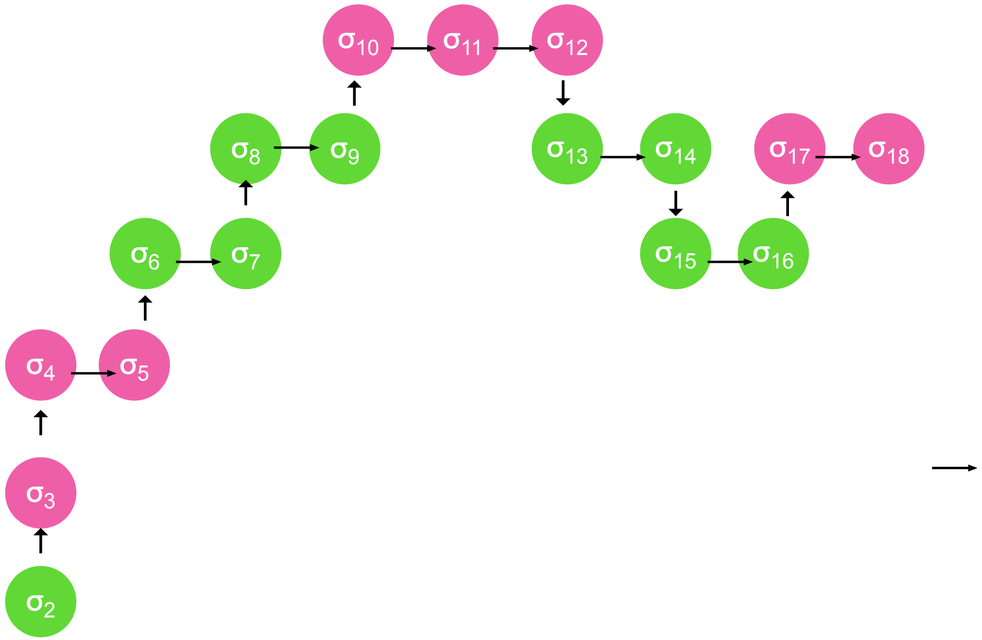

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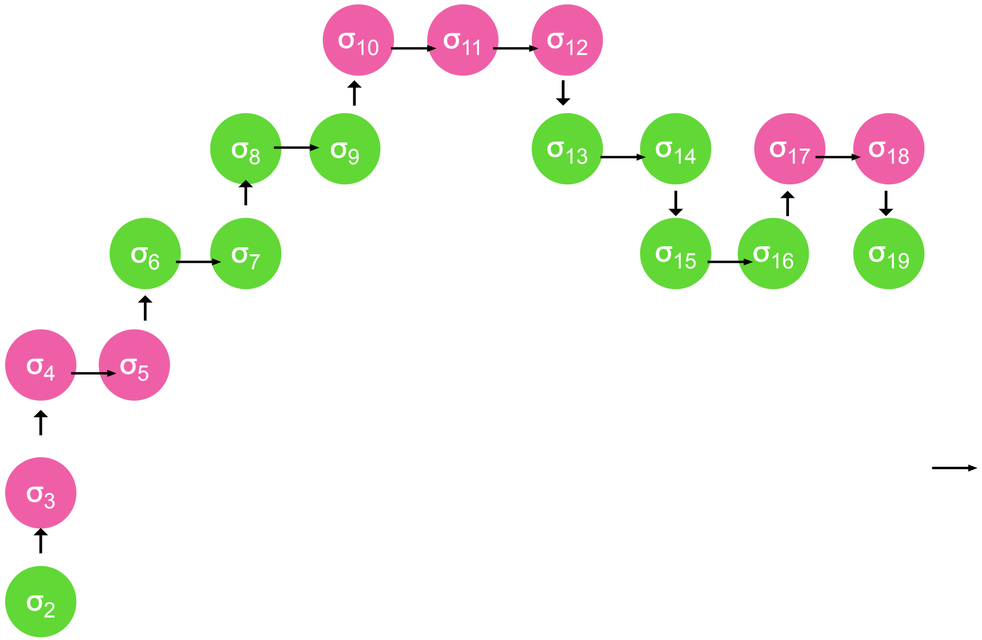

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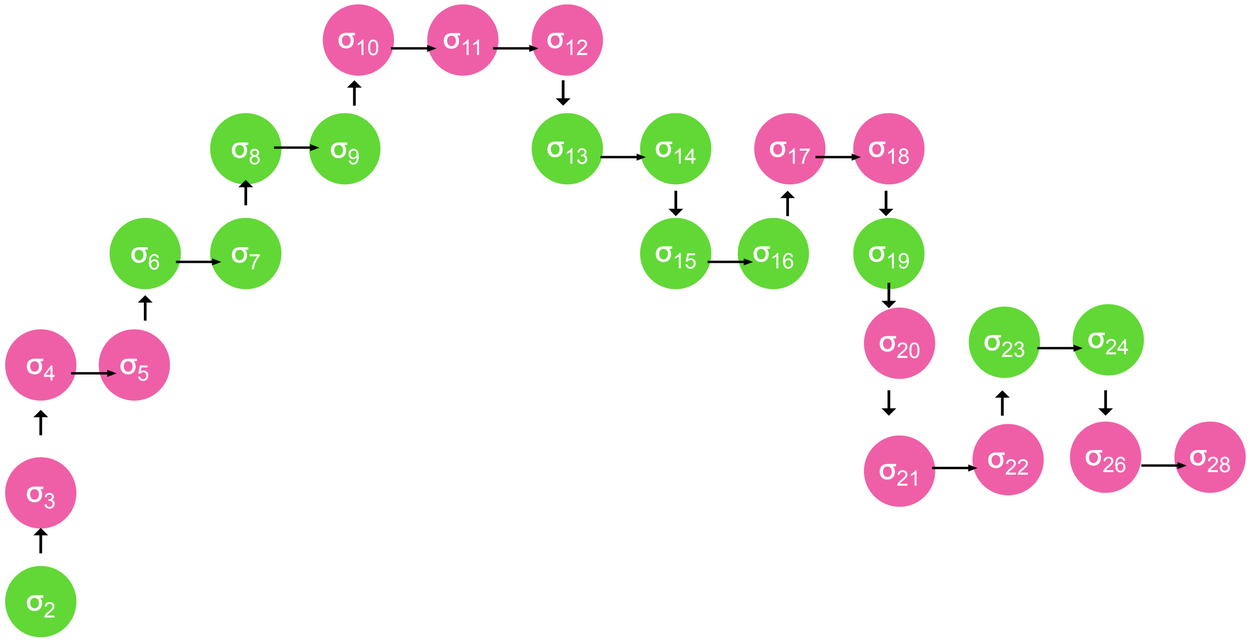

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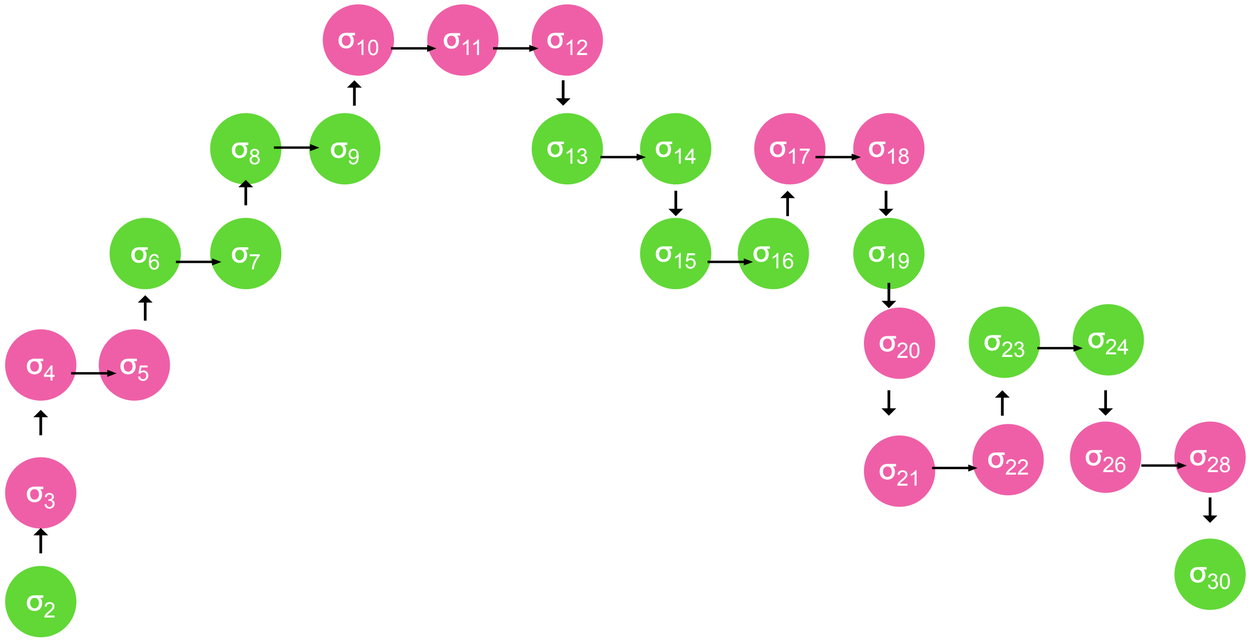

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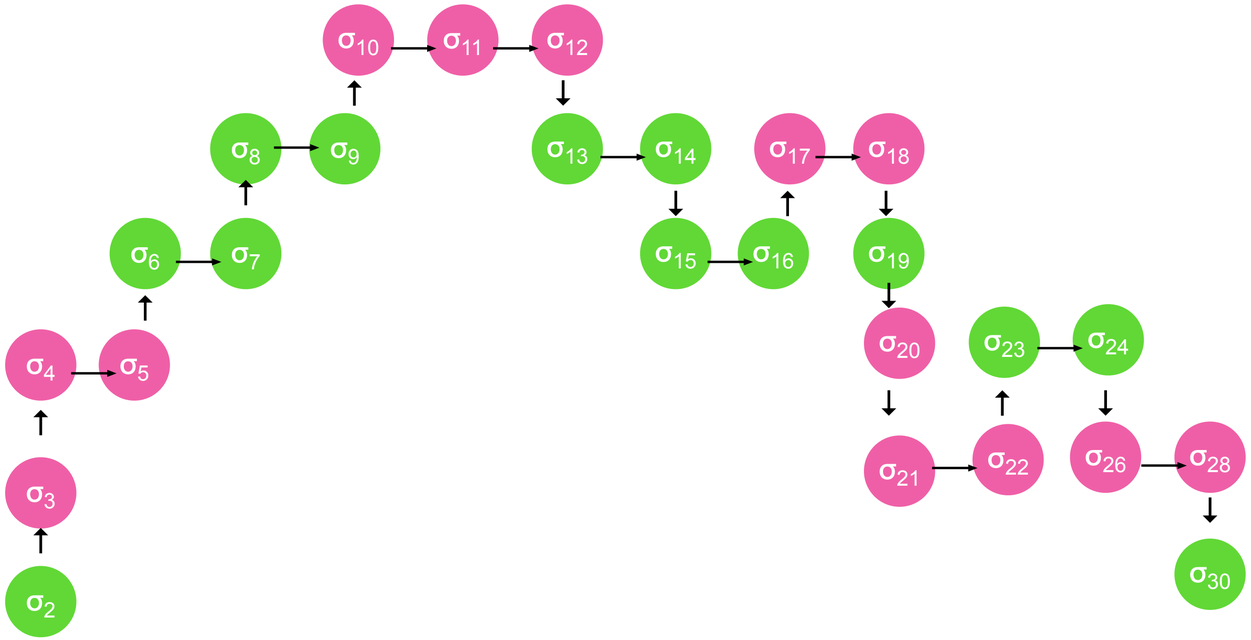

$\sigma_4 \text{ ->}^* \sigma_{18}$

$\sigma_4 \rightsquigarrow^* \sigma_{18}$

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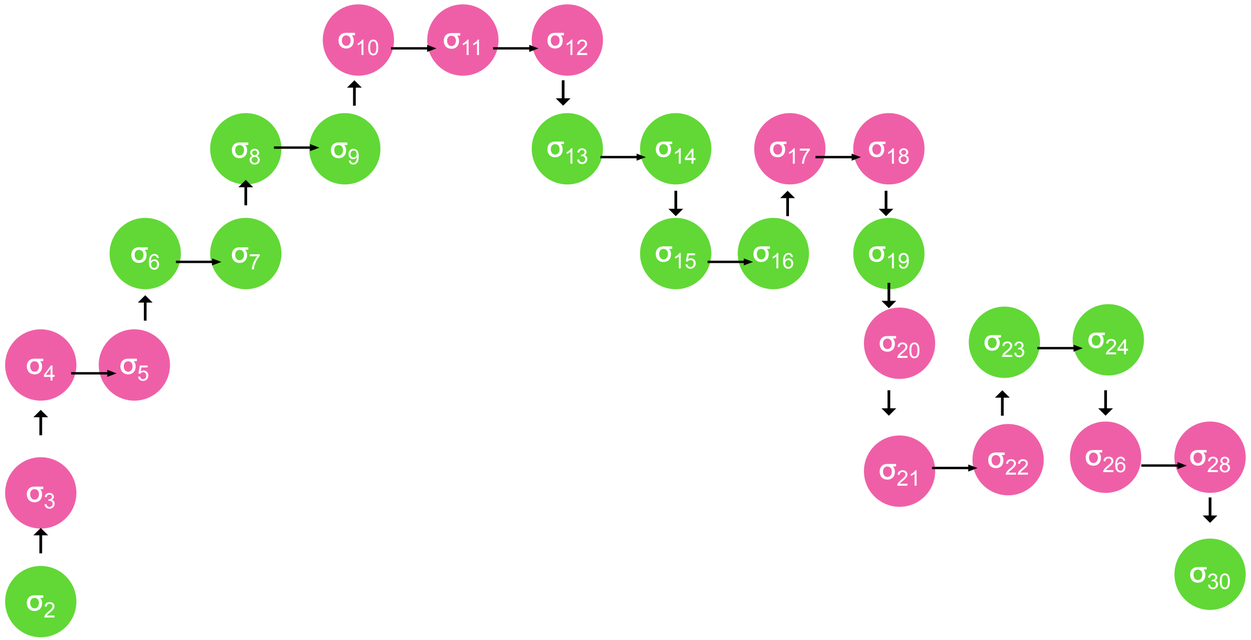

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$\sigma_4 \rightsquigarrow^* \sigma_{18}$

$\sigma_4 \to^* \sigma_{20}$

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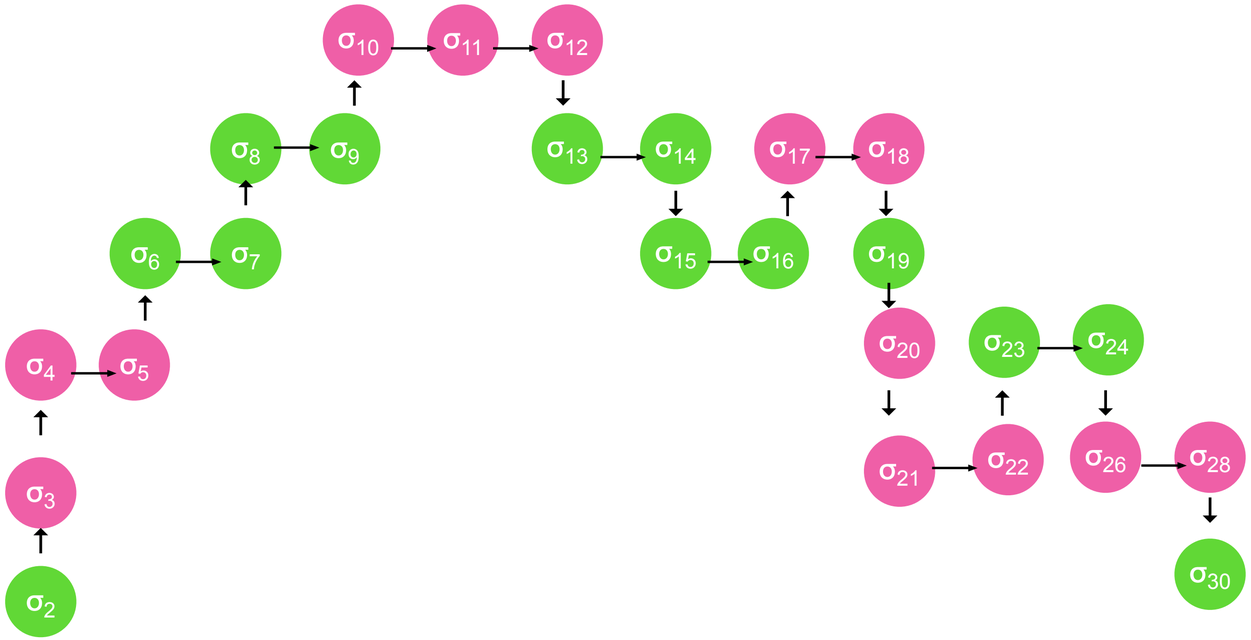

$\sigma_4 \text{ -}>^* \sigma_{18}$

$\sigma_4 \rightsquigarrow^* \sigma_{18}$

$\sigma_4 \text{ -}>^* \sigma_{20}$

$\sigma_4 \rightsquigarrow^* \sigma_{20}$

$\sigma_4 \text{ -}>^* \sigma_{21}$

$\neg(\sigma_4 \rightsquigarrow^* \sigma_{21})$

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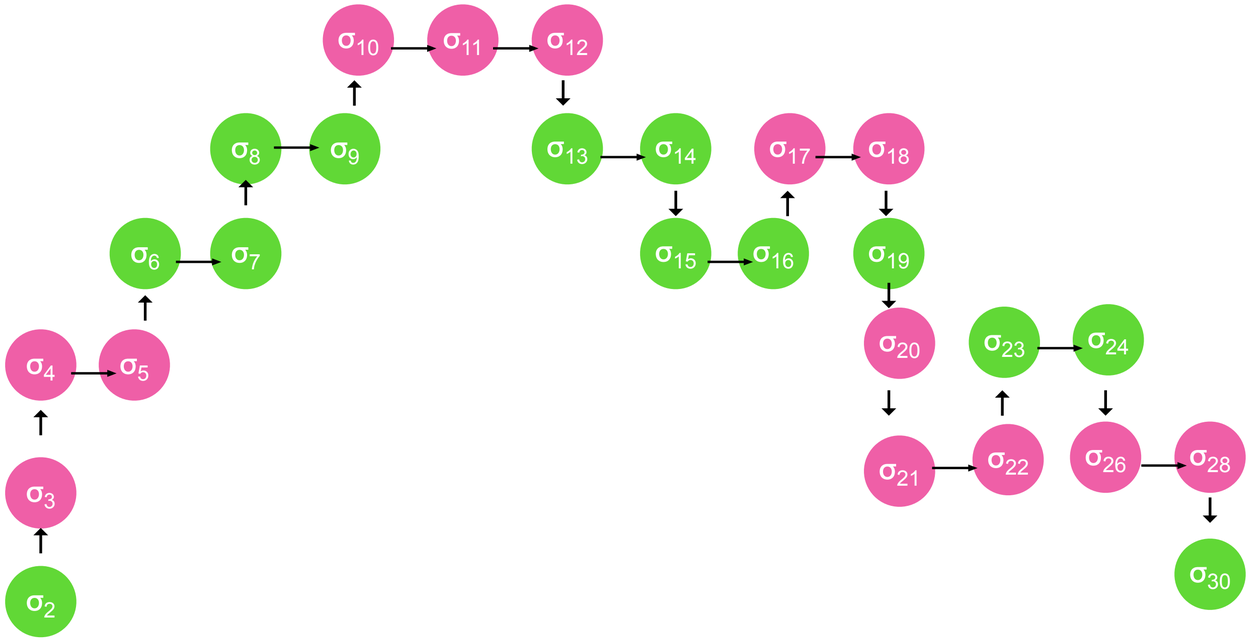

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$\sigma_4 \text{ -}>^* \sigma_{21}$

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$\sigma_4 \text{ -}>^* \sigma_{24}$

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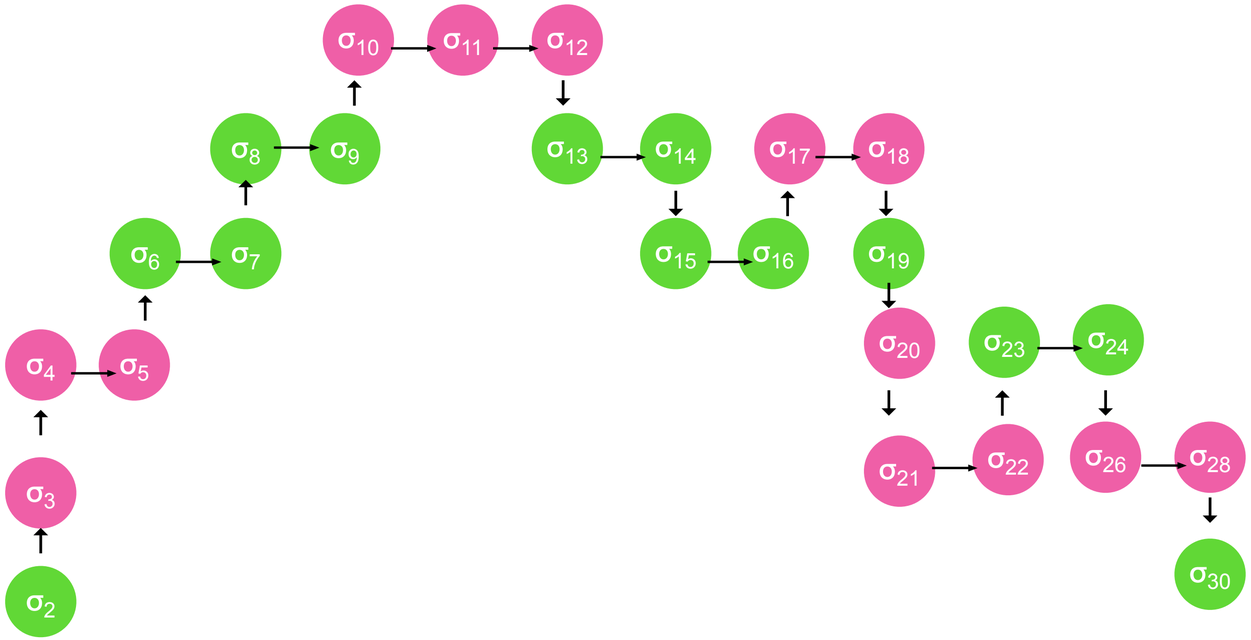
Therefore, $\forall$x1:C1,…xn:Cn { A } maens that



σ_4 -> * σ_{18}

$\sigma_4 \rightsquigarrow^* \sigma_{18}$

σ_4 -> * σ_{20}

$\sigma_4 \rightsquigarrow^* \sigma_{20}$

σ_4 -> * σ_{21}

$\neg(\sigma_4 \rightsquigarrow^* \sigma_{21})$

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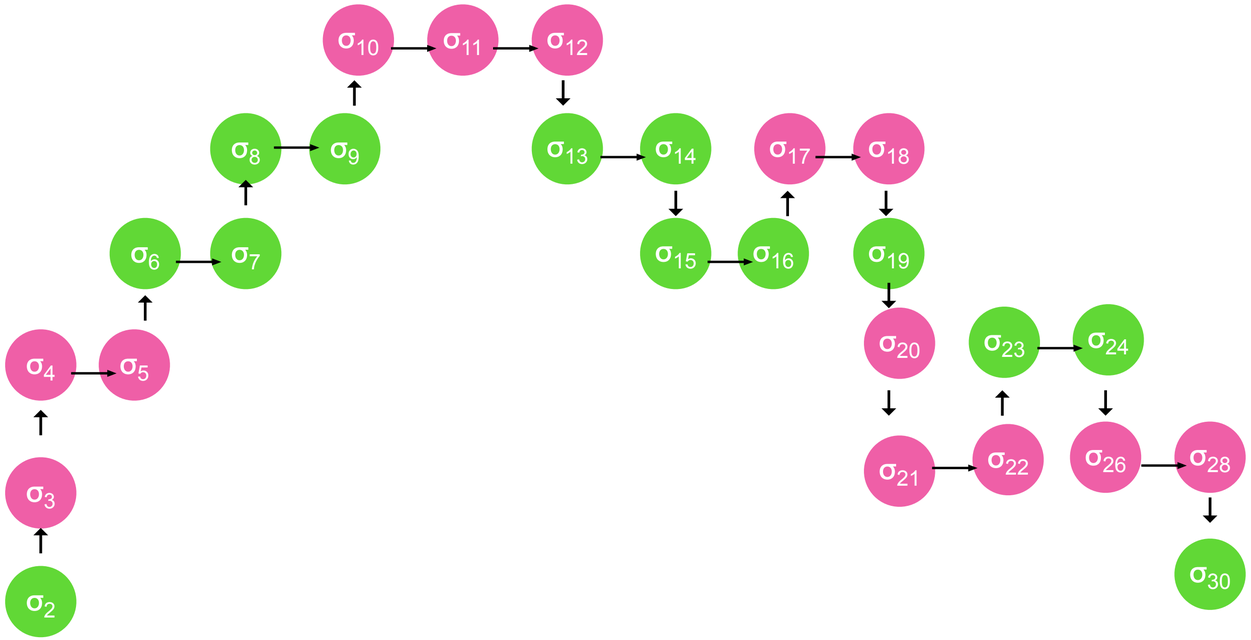

σ_4 ->* σ_{18}

$\sigma_4 \rightsquigarrow^* \sigma_{18}$

σ_4 ->* σ_{20}

$\sigma_4 \rightsquigarrow^* \sigma_{20}$ A preserved from σ_4 to σ_{20}

σ_4 ->* σ_{21}

$\neg(\sigma_4 \rightsquigarrow^* \sigma_{21})$

σ_4 ->* σ_{24}

$\neg(\sigma_4 \rightsquigarrow^* \sigma_{24})$

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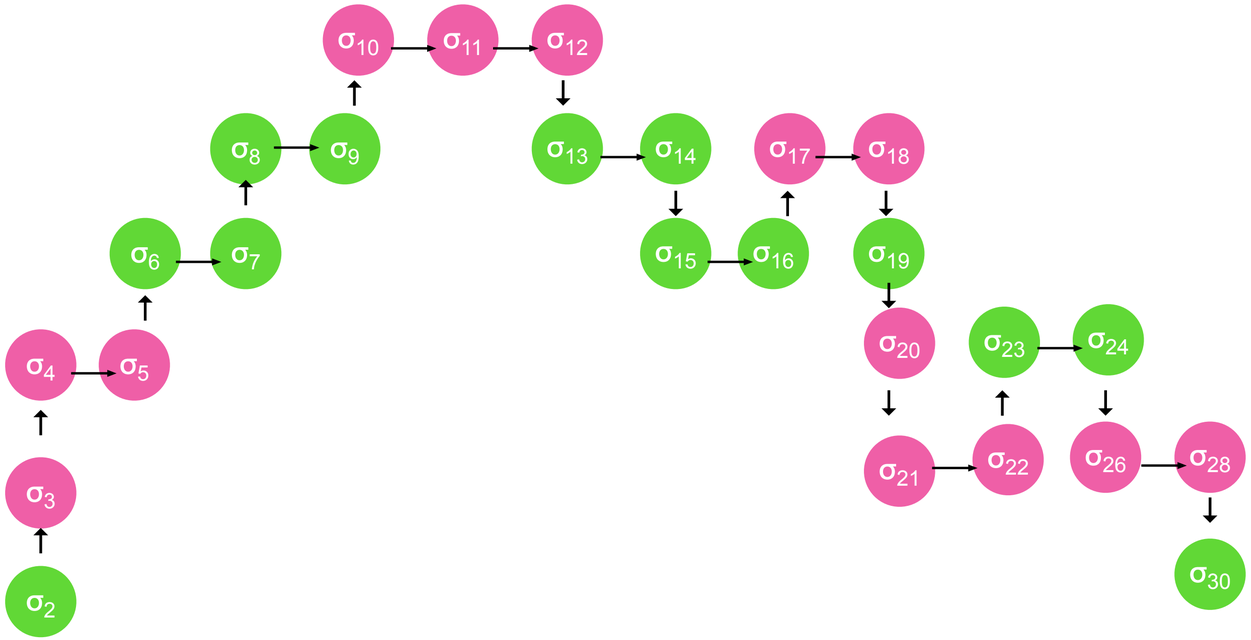

σ_4 ->* σ_{18}

$\sigma_4 \rightsquigarrow^* \sigma_{18}$ A preserved from σ_4 to σ_{18}

σ_4 ->* σ_{20}

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$\neg(\sigma_4 \rightsquigarrow^* \sigma_{21})$

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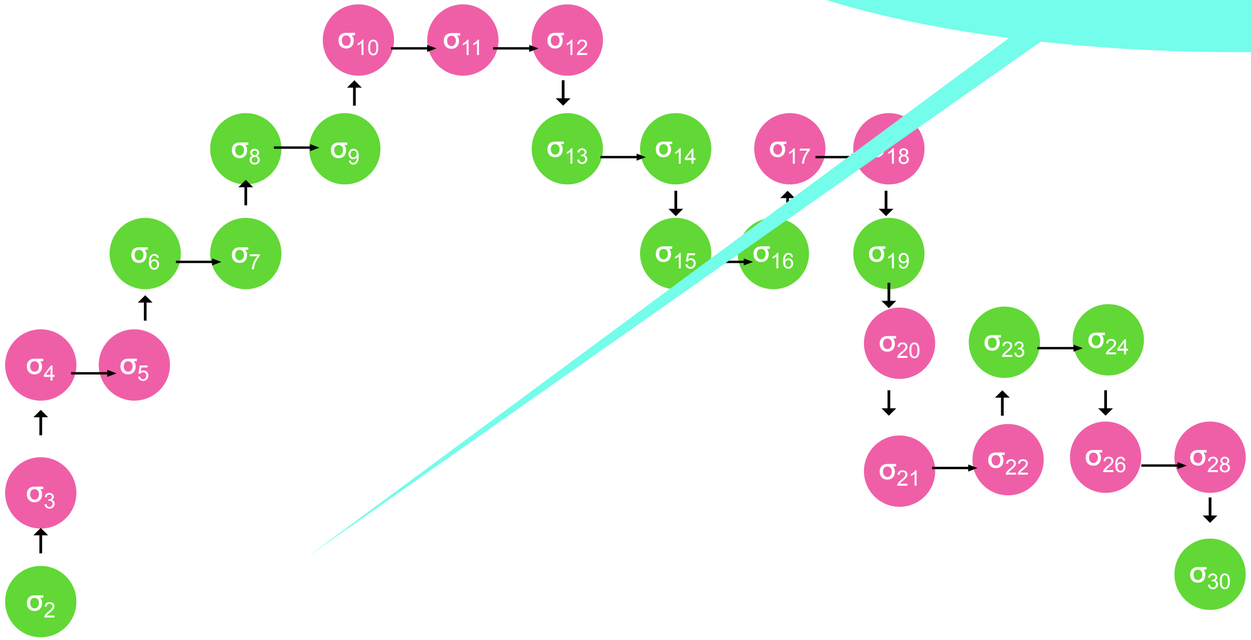
{ A } maens that

Execution is "relative" to a state



σ_{18} A preserved from σ_4 to σ_{18}

σ_4 ->* σ_{20}

$\sigma_4 \rightsquigarrow^* \sigma_{20}$ A preserved from σ_4 to σ_{20}

σ_4 ->* σ_{21}

$\neg(\sigma_4 \rightsquigarrow^* \sigma_{21})$

σ_4 ->* σ_{24}

$\neg(\sigma_4 \rightsquigarrow^* \sigma_{24})$

Challenge_1: A module spec S, such that

$M_{good} \models S$

$M_{bad} \not\models S$

$M_{better} \models S$

S1 $\triangleq$ $\forall$ a:Account. { ⟪a⟫ }

S2 $\triangleq$ $\forall$ a:Account. { ⟪a.pwd ⟫ }

S3 $\triangleq$ $\forall$ a:Account,b:Num. { ⟪a⟫ $\wedge$ a.blnce = b }

S4 $\triangleq$ $\forall$ a:Account, b:Num. { ⟪a.pwd ⟫ $\wedge$ a.blnce $\geq$ b }

Challenge_1: A module spec S, such that

$M_{good} \models S$

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API - agnostic:
a.blnce, a.pwd can be ghost

Talk about effects

Talk about
emergent behaviour

$S1 \triangleq \forall\ a{:}Account.\ \{ \langle\!\langle a \rangle\!\rangle \}$

$S2 \triangleq \forall\ a{:}Account.\ \{ \langle\!\langle a.pwd \rangle\!\rangle \}$

$S3 \triangleq \forall\ a{:}Account, b{:}Num.\ \{ \langle\!\langle a \rangle\!\rangle \wedge a.blnce = b \}$

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$M_{bad} \not\models S2$ $M_{bad} \not\models S4$

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$S4 \triangleq \forall\ a{:}Account, b{:}Num.\ \{ \langle\!\langle a.pwd \rangle\!\rangle \wedge a.blnce \geq b \}$

$M_{bad} \not\models S2$ $M_{bad} \not\models S4$ $M_{fine} \models S2$ $M_{fine} \models S4$

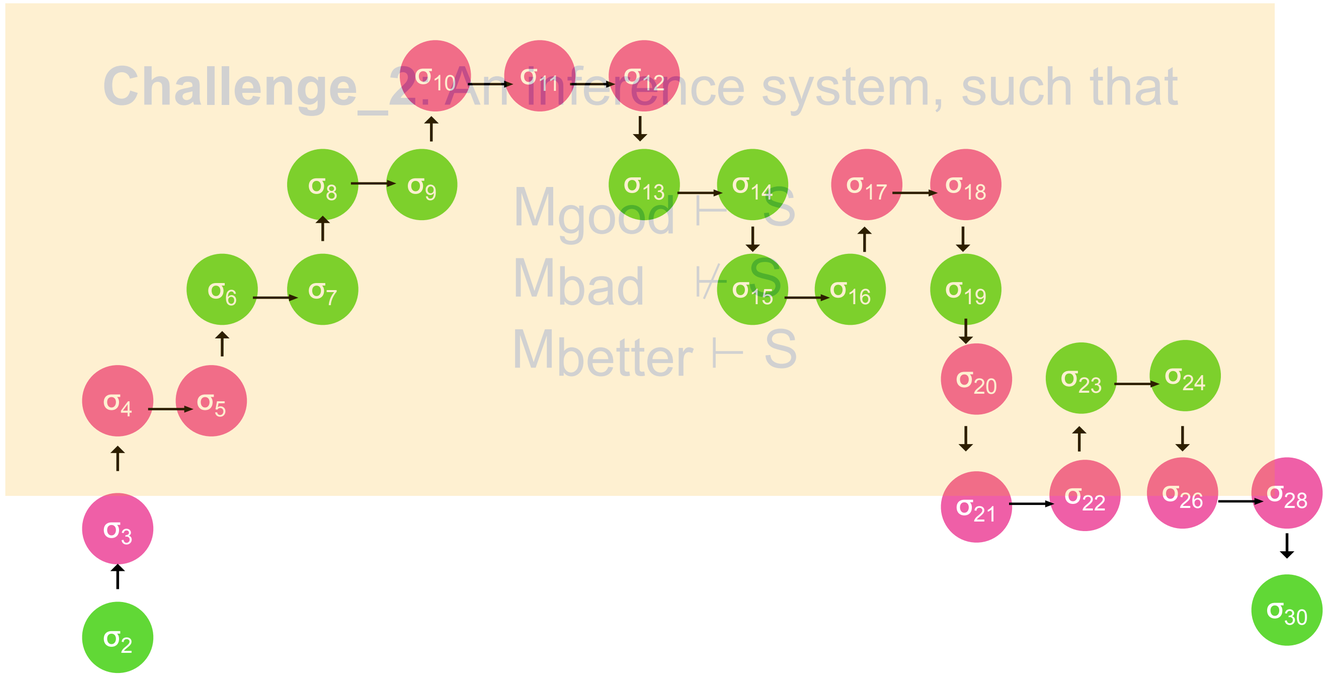
Challenge_2: An inference system, such that

$M_{good} \vdash S$

$M_{bad} \nvdash S$

$M_{better} \vdash S$

In the context of arbitrary, unlimited calls from internal to external,
and arbitrary, unlimited calls from external to internal,



Challenge_2: An inference system, such that …

An assertion A is **encapsulated** by module M, if
it can only be invalidated through calls to methods from M.

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For example:

$$\mathtt{Mod}_{\mathtt{bad}} \vDash \mathit{Encaps}(\mathtt{a:Account} \wedge \mathtt{a.balance} = \mathtt{bal})$$
$$\mathtt{Mod}_{\mathtt{better}} \vDash \mathit{Encaps}(\mathtt{a:Account} \wedge \mathtt{a.balance} = \mathtt{bal})$$

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An assertion A is **encapsulated** by module M, if
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Assume two further modules, Mod_{ul} and Mod_{pl}, which use ledgers to keep a map between accounts and their balances, which export functions that allow the update of this map. In Mod_{ul} the ledger is *not* protected, while in Mod_{pl} the ledger *is* protected.

$$Mod_{ul} \nvDash \mathit{Encaps}(\texttt{a : Account} \wedge \texttt{a.balance} = \texttt{bal})$$
$$Mod_{pl} \vDash \mathit{Encaps}(\texttt{a : Account} \wedge \texttt{a.balance} = \texttt{bal})$$

Challenge_2: An inference system, such that …

Three Stages

Assume an ulderlying Hoare logic of triples with usual meaning

$$M \vdash_{ul} \{ A \}\, s \,\{ A' \}$$

1st stage Expand it to Hoare logic of triples with usual meaning

$$M \vdash \{A\}\, s \,\{A'\},$$

satisfying the precon

2nd Stage Expand triples to quadruples

$$M \vdash \{A\}\, s \,\{A'\} \parallel \{A''\};$$

Which promises that

- termination of *s* leads to a state satisfying *A'*
- intermediate external states satisfy *A"*

3rd Stage Rules for module satisfying a specification

$$M \vdash S$$

Challenge_2: An inference system, such that …

1st stage

We expand underlying Hoare logic to Hoare logic of triples with usual meaning

$$\textbf{EXTEND}\quad \frac{M \vdash_{ul} \{ A \}\, s\, \{ A' \} \quad s \text{ contains no method call}}{M \vdash \{A\}\, s\, \{A'\}}$$

$$\textbf{TYPES-1}\quad \frac{s \text{ contains no method call}}{M \vdash \{x : C\}\, s\, \{x : C\}}$$

Challenge_2: An inference system, such that …

2nd stage

We expand triples to quadruples

$$\textbf{INV}\quad \frac{M \vdash \{A\}\, s\, \{A'\}}{M \vdash \{A\}\, s\, \{A'\} \parallel \{A''\}}$$

$$\textbf{TYPES-2}\quad \frac{M \vdash \{A\}\, s\, \{A'\} \parallel \{A''\}}{M \vdash \{x : C \wedge A\}\, s\, \{x : C \wedge A'\} \parallel \{A''\}}$$

$$\textbf{COMBINE}\quad \frac{M \vdash \{A_1\}\, s\, \{A_2\} \parallel \{A\} \qquad M \vdash \{A_3\}\, s\, \{A_4\} \parallel \{A\}}{M \vdash \{A_1 \wedge A_3\}\, s\, \{A_2 \wedge A_4\} \parallel \{A\}}$$

$$\textbf{SEQU}\quad \frac{M \vdash \{A_1\}\, s_1\, \{A_2\} \parallel \{A\} \qquad M \vdash \{A_2\}\, s_2\, \{A_3\} A}{M \vdash \{A_1\}\, s_1; s_2\, \{A_3\} \parallel \{A\}}$$

$$\textbf{CONSEQU}\quad \frac{M \vdash \{A_2\}\, s\, \{A_3\} \parallel \{A_4\} \qquad M \vdash A_1 \rightarrow A_2 \qquad M \vdash A_3 \rightarrow A_5 \qquad M \vdash A_4 \rightarrow A_6}{M \vdash \{A_2\}\, s\, \{A_5\} \parallel \{A_6\}}$$

Challenge_2: An inference system, such that …

3rd stage

$$\frac{M \vdash \mathscr{S}pec(M)}{\vdash M}\ \textsc{WellFrm_Mod} \qquad \frac{M \vdash S_1 \qquad M \vdash S_2}{M \vdash S_1 \wedge S_2}\ \textsc{Comb_Spec}$$

Challenge_2: An inference system, such that …

3rd stage

$$\frac{???}{M \vdash \forall \overline{x : C}\{A\}}\ \textsc{invariant}$$

Challenge_2: An inference system, such that …

$$\frac{M \vdash Encps(\overline{x : C} \wedge A)}{M \vdash \forall \overline{x : C}\{A\}}\ \textbf{INVARIANT}$$

Challenge_2: An inference system, such that …

$$\frac{M \vdash Encps(\overline{x : C} \wedge A) \qquad \forall D, m : \quad \texttt{mBody}(m, D, M) = \texttt{public}\ (\overline{y : C})\{\ stmt\ \} \implies}{M \vdash \forall \overline{x : C}\{A\}}\ \textbf{INVARIANT}$$

Challenge_2: An inference system, such that …

$$\frac{\begin{array}{l} M \vdash Encps(\overline{x : C} \wedge A) \\ \forall D, m : \quad \texttt{mBody}(m, D, M) = \texttt{public}\ (\overline{y : C})\{\ stmt\ \} \implies \\ \qquad M \vdash \{\ \texttt{this} : \texttt{D}, \overline{y : D}, \overline{x : C} \wedge PRE(A)\ \}\ stmt\ \{\ POST(A)\ \} \parallel \{\ POST(A)\ \} \end{array}}{M \vdash \forall \overline{x : C}\{A\}}\ \textbf{INVARIANT}$$

Remit_3: An inference system, such that we can prove external calls

Challenge_4: An inference system, such we can prove external calls

$$\frac{}{M \vdash \{\ y_0 : \texttt{ext} \quad , \quad \} \ u := y_0.m(y_1, ..y_n)\ \{\ ???\ \} \parallel \{\ ??\ \}}\ [\textsc{Call\_Ext}]$$

Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{ \; y_0 : \texttt{ext} \quad \overline{\quad\quad} \quad , \quad\quad\quad\quad \} \; u := y_0.m(y_1, ..y_n) \; \{ \quad ??? \quad \} \; \| \; \{ \; ?? \; \}} \; [\textsc{Call\_Ext}]$$

Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{ \; y_0 : \texttt{ext} \wedge \overline{x : D} \; , \qquad \} \; u := y_0.m(y_1, ..y_n) \; \{ \quad ??? \quad \} \parallel \{ \; ?? \; \}} \; [\textsc{Call\_Ext}]$$

Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{ \; y_0 : \texttt{ext} \wedge \overline{x : D} \wedge A \multimap \forall \overline{y} \; \} \, u := y_0.m(y_1, ..y_n) \, \{ \quad ??? \quad \} \parallel \{ \; ?? \; \}} \; [\textsc{Call\_Ext}]$$

Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{ \; y_0 : \texttt{ext} \wedge \overline{x : D} \wedge A \multimap \forall \overline{y} \; \} \; u := y_0.m(y_1, ..y_n) \; \{ \; A \multimap \forall \overline{y} \; \} \; \| \; \{ \; ?? \; \}} \; [\textsc{Call\_Ext}]$$

Challenge_4: An inference system, such we can prove external calls

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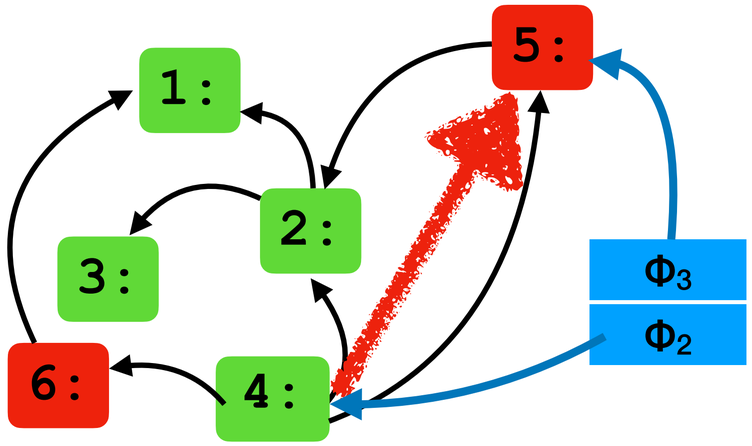
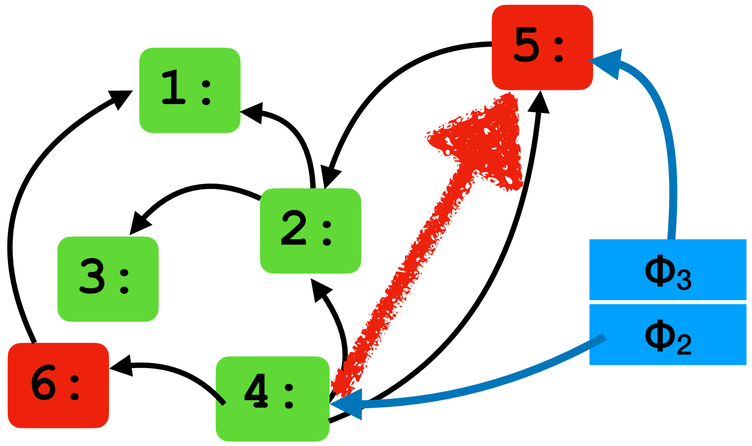
Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{ \; y_0 : \texttt{ext} \wedge \overline{x : D} \wedge A \multimap \forall \overline{y} \; \} \, u := y_0.m(y_1, ..y_n) \, \{ \; A \multimap \forall \overline{y} \; \} \; \| \; \{ \; A \; \}} \; [\textsc{Call\_Ext}]$$



Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{\; y_0 : \texttt{ext} \wedge \overline{x : D} \wedge A \multimap \nabla \overline{y} \;\}\; u := y_0.m(y_1, ..y_n) \;\{\; A \multimap \nabla \overline{y} \;\} \;\|\; \{\; A \;\}} \; [\textsc{Call\_Ext}]$$

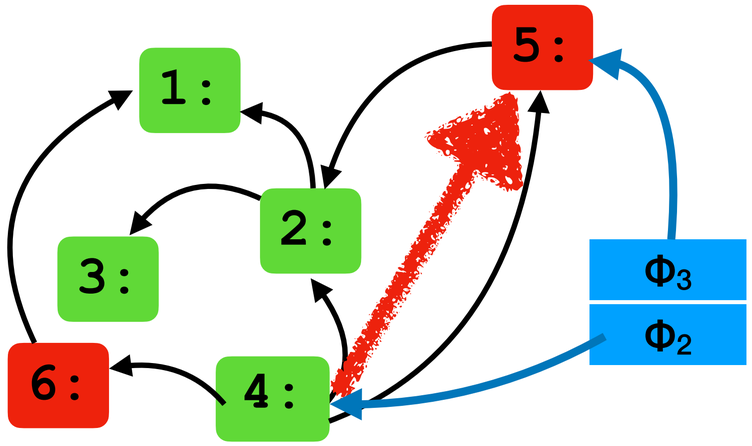


Definition 5.7. [The $\multimap\nabla$ operator] is defined below

$$\begin{array}{rcl@{\qquad}rcl}
(\langle e \rangle) \multimap \nabla \overline{y} & \triangleq & \langle e \rangle \leftrightarrowtail \overline{y} & (A_1 \wedge A_2) \multimap \nabla \overline{y} & \triangleq & (A_1 \multimap \nabla \overline{y}) \wedge (A_2 \multimap \nabla \overline{y}) \\
(\langle e \rangle \leftrightarrowtail \overline{u}) \multimap \nabla \overline{y} & \triangleq & \langle e \rangle \leftrightarrowtail \overline{u} & (\forall x : C.A) \multimap \nabla \overline{y} & \triangleq & \forall x : C.(A \multimap \nabla \overline{y}) \\
(\,\mathbf{intl}\, e\,) \multimap \nabla \overline{y} & \triangleq & \mathbf{intl}\, e & (\neg A) \multimap \nabla \overline{y} & \triangleq & \neg (A \multimap \nabla \overline{y}) \\
e \multimap \nabla \overline{y} & \triangleq & e & (e : C) \multimap \nabla \overline{y} & \triangleq & e : C
\end{array}$$

Challenge_4: An inference system, such we can prove external calls

$$\frac{\vdash \; M \; : \; \forall \overline{x : D}\{A\}}{M \vdash \{\; y_0 : \texttt{ext} \wedge \overline{x : D} \wedge A \multimap \nabla \overline{y} \;\}\; u := y_0.m(y_1, ..y_n) \;\{\; A \multimap \nabla \overline{y} \;\} \;\|\; \{\; A \;\}} \; [\textsc{Call\_Ext}]$$



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e \multimap \nabla \overline{y} & \triangleq & e & (e : C) \multimap \nabla \overline{y} & \triangleq & e : C
\end{array}$$

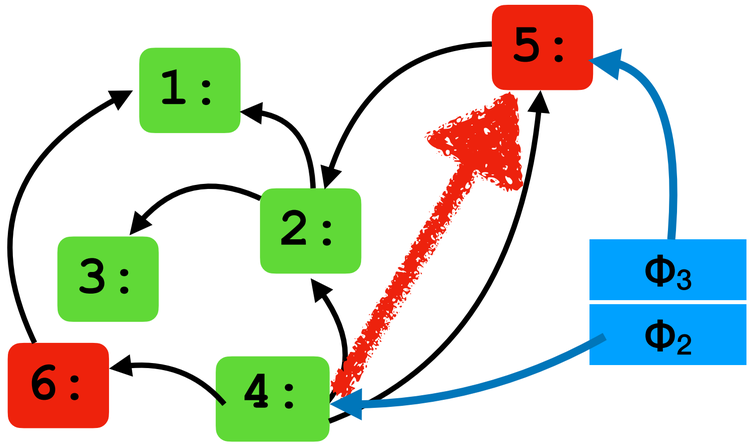
Lemma 5.8. For any state σ, assertion A, and variables $\overline{y}, \overline{z}$, disjoint with one another: If $fv(A) = \emptyset$, then

$$\begin{array}{lcl}
(1)\;\; M, \sigma \models A \multimap \nabla \overline{y} & \Longrightarrow & M,\; \sigma \nabla \overline{y} \models A \\
(2)\;\; M, \sigma \nabla (\overline{y}, \overline{z}) \models A & \Longrightarrow & M, \sigma \models A \multimap \nabla \overline{y}
\end{array}$$

Challenge_4: An inference system, such we can prove external calls

Protection is
"relative" to a frame;
Our $-\!\!\triangledown$ operator
helps us switch to callee's view

$$M \vdash \{\ y_0 : \texttt{ext} \wedge \overline{x : D} \wedge A \mathrel{-\!\!\triangledown} \overline{y}\ \}\ u := y_0 \ldots$$



Definition 5.7. [The $-\!\!\triangledown$ operator] is defined below

$$\begin{array}{rcl@{\qquad}rcl}
(\langle e \rangle) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & \langle e \rangle \leftrightarrow\!\!\!\!\times\ \overline{y} & (A_1 \wedge A_2) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & (A_1 \mathrel{-\!\!\triangledown} \overline{y}) \wedge (A_2 \mathrel{-\!\!\triangledown} \overline{y}) \\
(\langle e \rangle \leftrightarrow\!\!\!\!\times\ \overline{u}) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & \langle e \rangle \leftrightarrow\!\!\!\!\times\ \overline{u} & (\forall x : C.A) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & \forall x : C.(A \mathrel{-\!\!\triangledown} \overline{y}) \\
(\ \mathbf{intl}\ e\ ) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & \mathbf{intl}\ e & (\neg A) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & \neg (A \mathrel{-\!\!\triangledown} \overline{y}) \\
e \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & e & (e : C) \mathrel{-\!\!\triangledown} \overline{y} & \triangleq & e : C
\end{array}$$

Lemma 5.8. For any state σ, assertion A, and variables $\overline{y}, \overline{z}$, disjoint with one another: If $fv(A) = \emptyset$, then

$$\begin{array}{lcl}
(1)\ M, \sigma \models A \mathrel{-\!\!\triangledown} \overline{y} & \Longrightarrow & M,\ \sigma \triangledown \overline{y} \models A \\
(2)\ M, \sigma \triangledown (\overline{y}, \overline{z}) \models\ A & \Longrightarrow & M, \sigma \models A \mathrel{-\!\!\triangledown} \overline{y}
\end{array}$$

- Distinction between external/internal objects

- ∀ x: … { A } Two state invariants for external states / relative execution

- Specifications talk about necessary conditions for effect:

  ∀ x: … { ⟪e ⟫ ∧ A }

  means that **capability** e is needed in order to invalidate A

- ⟪e ⟫: capability e is protected from reachable external objects

Summary

- API-agrnositc spec,
- “Algorithmic” inference system system,
- Reason with open calls

- Protection, ⟪e ⟫, is relative to state σ. Use —▽ to switch view
- Execution, M’*M, σ ↝* σ’ , is relative to state σ

- Surpises:
  - Use sufficient conditions to talk about necessary conditions
  - from temporal operators/logics to invariants/Hoare logics