INTO-CPS

Isabelle/UTP

A Verification Toolbox for Unifying Theories

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INTO-CPS

into-cps.au.dk

Horizon 2020 Programme

VeTSS Project

"Mechanised Assume-Guarantee Reasoning for Control Law Diagrams via Circus"

- AG proof support for discrete time Simulink diagrams
- Circus: stateful reactive language extending CSP
- use of reactive contracts to specify properties
- develop a library of examples and two case studies
- mechanised proof support for Simulink in Isabelle/UTP
- researcher: Dr. Kangfeng Ye (Randall)

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- use alphabetised relational calculus as a *lingua franca*
- programs-as-predicates: specification + implementation
- link different semantic models (operational, axiomatic etc.)
- build verification tools for various paradigms

Example: Operational Semantics and Hoare Calculus

Definition (Transition Relation)

$$(\sigma_1, P_1) \to (\sigma_2, P_2) \triangleq \langle \sigma_1 \rangle ; P_1 \sqsubseteq \langle \sigma_2 \rangle ; P_2$$

Theorem (Operational Laws)

$$\frac{(\sigma, P) \to (\rho, Q)}{(\sigma, P ; R) \to (\rho, Q ; R)} \text{ SEQ-STEP}$$

$$\frac{\sigma \models c}{(\sigma, \textbf{\textit{if}}\ c\ \textbf{\textit{then}}\ P\ \textbf{\textit{else}}\ Q) \to (\sigma, P)} \text{ COND-TRUE}$$

$$\frac{}{(\sigma, x := v) \to (\sigma(x := \sigma \dagger v), \mathit{II})} \text{ ASSIGN}$$

$$\frac{\sigma \models c}{(\sigma, \textbf{\textit{while}}\ c\ \textbf{\textit{do}}\ P) \to (\sigma, P ; \textbf{\textit{while}}\ c\ \textbf{\textit{do}}\ P)} \text{ ITER-COPY}$$

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Theorem (Linking)

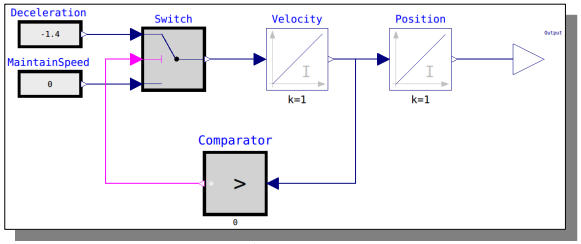
$$\{ p \}\, Q \,\{ r \} \Leftrightarrow \frac{\sigma_1 \models p \quad (\sigma_1, Q) \rightarrow (\sigma_2, \mathit{II})}{\sigma_2 \models r}$$

- operators are denotations, laws are theorems
- we apply this technique to more complex computational paradigms, such as concurrent and hybrid systems

INTO-CPS Multi-Modelling

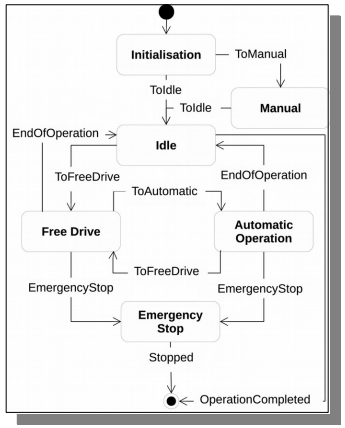
$$\dot{\psi} = \frac{(\omega_r - \omega_l)}{T_c}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = V_c \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \end{bmatrix}$$

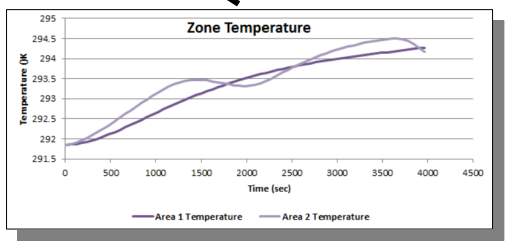



```c
#include <stdio.h>
#define MAX 10
int main()
{
    char array[MAX][MAX], c = 0;
    int d = 1, x = 0, i, j;

    do
        scanf("%s", array[x]);
    while (array[x++][0] != '0');

    {
        float* pf;
        int xx, *pi = (int*)&array[0][7];
        xx = ((*pi) & 0x41000000);
        pf = (float*)&xx;
        printf("%5.2f\n", *pf);
    }
```

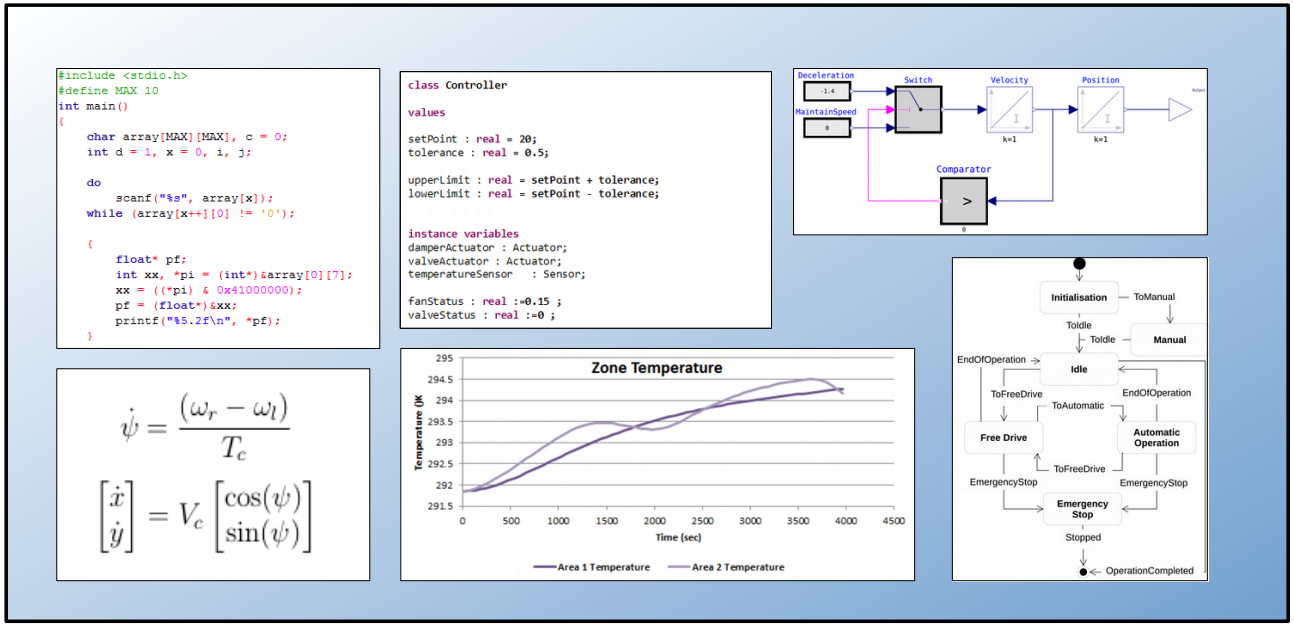
UTP Multi-Model Semantics




```
class Controller

values

setPoint : real = 20;
tolerance : real = 0.5;

upperLimit : real = setPoint + tolerance;
lowerLimit : real = setPoint - tolerance;


instance variables
damperActuator : Actuator;
valveActuator : Actuator;
temperatureSensor    : Sensor;

fanStatus : real :=0.15 ;
valveStatus : real :=0 ;
```



Vision: UTP CPS Verification Foundations



$$\dot{\psi} = \frac{(\omega_r - \omega_l)}{T_c}$$

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Graphical Notations	Object Orientation	Differential Equations	Concurrency	Contracts
Hybrid Systems	State	Real-time	Reactive Systems	

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- define syntax for programs and create verification calculi
- via formalisation of semantic “building blocks” (UTP theories)
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- a verification toolbox for the UTP based on Isabelle/HOL
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- define syntax for programs and create verification calculi
- via formalisation of semantic “building blocks” (UTP theories)
- utilise Isabelle’s powerful proof automation for verification
- formalise links between domains using Galois connection
- large library of formalised algebraic laws of programming

Examples

```
lemma hoare_ex_1:
  "{true}(z := &x) ◁ (&x ≥u &y) ▷r (z := &y){&z =u maxu(&x, &y)}u"
  by (hoare_auto)

lemma hoare_ex_2:
  assumes "X > 0" "Y > 0"
  shows
  "{&x =u «X» ∧ &y =u «Y»}
    while ¬(&x =u &y)
    invr &x >u 0 ∧ &y >u 0 ∧ (gcdu(&x,&y) =u gcdu(«X»,«Y»))
    do
       (x := (&x - &y)) ◁ (&x >u &y) ▷r (y := (&y - &x))
    od
    {&x =u gcdu(«X», «Y»)}u"
  using assms by (hoare_auto, (metis gcd.commute gcd_diff1)+)
```

Examples

```
definition Pay :: "index ⇒ index ⇒ money ⇒ action_mdx" where
"Pay i j n =
  pay.((«i»,«j»,«n»)ᵤ) ⟶
    ((reject.(«i») ⟶ Skip)
      ◁ «i» =ᵤ «j» ∨ «i» ∉ᵤ domᵤ(&accts) ∨ «n» ≤ᵤ 0 ∨ «n» >ᵤ &accts(«i»)ₐ ▷R
    ({accts[«i»]} :=C (&accts(«i»)ₐ - «n») ;;
     {accts[«j»]} :=C (&accts(«j»)ₐ + «n») ;;
     accept.(«i») ⟶ Skip))"

definition PaySet :: "index ⇒ (index × index × money) set" where
[upred_defs]: "PaySet cardNum = {(i,j,k). i < cardNum ∧ j < cardNum ∧ i ≠ j}"

definition AllPay :: "index ⇒ action_mdx" where
"AllPay cardNum = (⨅ (i, j, n) ∈ PaySet cardNum • Pay i j n)"
```

Examples

```
theorem money_constant:
  assumes "finite cards" "i ∈ cards" "j ∈ cards" "i ≠ j"
  shows "[dom_u(&accts) =_u «cards» ⊢ true | sum_u($accts) =_u sum_u($accts´)]_C ⊑ Pay i j n"
-- {* We first calculate the reactive design contract and apply refinement introduction *}
proof (simp add: assms Pay_contract, rule CRD_refine_rdes)

  -- {* Three proof obligations result for the pre/peri/postconditions. The first requires us to
    show that the contract's precondition is weakened by the implementation precondition.
    It is because the implementation's precondition is under the assumption of receiving an
    input and the money amount constraints. We discharge by first calculating the precondition,
    as done above, and then using the relational calculus tactic. *}

  from assms
  show "`[dom_u(&accts) =_u «cards»]_S< ⇒
          I(true,⟨(pay·(«i», «j», «n»)_u)_u⟩) ⇒_r
          [(«i» ∉_u dom_u(&accts) ∨ «n» ≤_u 0 ∨ &accts(«i»)_a <_u «n») ∨
            «i» ∈_u dom_u(&accts) ∧ «j» ∈_u dom_u(&accts)]_S<`"
    by (rel_auto)
```

Examples

```
theorem extChoice_commute:
  assumes "P is NCSP" "Q is NCSP"
  shows "P □ Q = Q □ P"
  by (rdes_eq cls: assms)

theorem extChoice_assign:
  assumes "P is NCSP" "Q is NCSP"
  shows "x :=C v ;; (P □ Q) = (x :=C v ;; P) □ (x :=C v ;; Q)"
  by (rdes_eq cls: assms)

theorem stop_seq:
  assumes "P is NCSP"
  shows "Stop ;; P = Stop"
  by (rdes_eq cls: assms)
```

Examples

```
definition
"BrakingTrain =
   c:accel, c:vel, c:pos := «normal_deceleration», «max_speed», «0» ;;
   ⟨{&accel,&vel,&pos} • «train_ode»⟩ₕ untilₕ ($vel´ ≤ᵤ 0) ;; c:accel := 0"

theorem braking_train_pos_le:
 "($c:accel´ =ᵤ 0 ∧ ⌈$pos´ <ᵤ 44⌉ₕ) ⊑ BrakingTrain" (is "?lhs ⊑ ?rhs")
proof -
  -- {* Solve ODE, replacing it with an explicit solution: @{term train_sol}. *}
  have "?rhs =
    c:accel, c:vel, c:pos := «-1.4», «4.16», «0» ;;
    {&accel,&vel,&pos} ←ₕ «train_sol»(&accel,&vel,&pos)ₐ(«time»)ₐ untilₕ ($vel´ ≤ᵤ 0) ;;
    c:accel := 0"
  by (simp only: BrakingTrain_def train_sol)
  -- {* Set up initial values for the ODE solution using assigned variables. *}
  also have "... =
    {&accel,&vel,&pos} ←ₕ «train_sol(-1.4,4.16,0)(time)» untilₕ ($vel´ ≤ᵤ 0) ;; c:accel := 0"
  by (simp add: assigns_r_comp usubst unrest alpha, literalise, simp)
```

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- Isabelle/UTP: http://www.cs.york.ac.uk/~simonf/utp-isabelle
- GitHub: https://github.com/isabelle-utp/utp-main
- Email: simon.foster@york.ac.uk

References

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